PAE, Info, Syl

DÚ 11 - ŘEŠENÍ

DIFERENCIÁLNÍ ROVNICE 1. ŘÁDU

1) $9x \cdot y' + 9y = \frac{7}{\cos^2 x}$

$$a \cdot \ln x = \ln x^a$$
$$\ln a + \ln b = \ln(a \cdot b)$$
$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$
$$y' \sim \frac{dy}{dx}$$
$$k' \sim \frac{dk}{dx}$$
$$C = \ln K$$

$$9x \cdot y' + 9y = 0$$
$$9x \cdot \frac{dy}{dx} + 9y = 0$$
$$9x \cdot \frac{dy}{dx} = -9y$$
$$\frac{dy}{y} = -\frac{\cancel{9}dx}{\cancel{9}x}$$
$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$
$$\ln|y| = -\ln|x| + C$$
$$\ln|y| = \ln|x^{-1}| + \ln K$$
$$\cancel{\ln}|y| = \cancel{\ln}|x^{-1} \cdot K|$$
$$y = x^{-1} \cdot K$$

$$y = x^{-1} \cdot k(x)$$
$$y' = -x^{-2}k(x) + x^{-1}k'(x)$$

$$9x\left(-x^{-2}k(x) + x^{-1}k'(x)\right) + 9x^{-1}k(x) = \frac{7}{\cos^2 x}$$
$$-9\cancel{x^{-1}k(x)} + 9k'(x) + 9\cancel{x^{-1}k(x)} = \frac{7}{\cos^2 x}$$
$$9k'(x) = \frac{7}{\cos^2 x}$$
$$9\frac{dk}{dx} = \frac{7}{\cos^2 x}$$
$$\int dk = \int \frac{7\,dx}{\cos^2 x \cdot 9}$$
$$k = \frac{7}{9}\int \frac{dx}{\cos^2 x}$$
$$k = \frac{7}{9} \cdot \operatorname{tg} x + C$$

$$y = x^{-1} \cdot \left(\frac{7}{9}\operatorname{tg} x + C\right)$$
$$y = \frac{\frac{7}{9}\operatorname{tg} x + C}{x}$$
$$y = \frac{7\operatorname{tg} x + C}{9x}$$

2) $2y' + 20y = 3e^{-6x}$

Převod čísla na ln:

$a = \ln e^{a}$

$7 = \ln e^{7}$

$3x = \ln e^{3x}$ atd...

$$2\frac{dy}{dx} + 20y = 0$$

$$2\frac{dy}{dx} = -20y$$

$$\int\frac{dy}{y} = -\int\frac{20\,dx}{2}$$

$$\ln|y| = -10\int dx$$

$$\ln|y| = -10x + C$$

$$\ln|y| = \ln e^{-10x} + \ln K$$

$$\ln|y| = \ln(e^{-10x}\cdot K)$$

$$y = e^{-10x}\cdot K$$

$$y = e^{-10x}\cdot k(x)$$

$$y' = e^{-10x}\cdot(-10)\cdot k(x) + e^{-10x}\cdot k'(x)$$

$$2\left[e^{-10x}\cdot(-10)\cdot k(x) + e^{-10x}\cdot k'(x)\right] + 20\cdot e^{-10x}k(x) = 3e^{-6x}$$

$$\cancel{-20e^{-10x}}\cdot k(x) + 2e^{-10x}\cdot k'(x) + \cancel{20e^{-10x}}k(x) = 3e^{-6x}$$

$$2e^{-10x}\frac{dk}{dx} = 3e^{-6x}$$

$$\int dk = \int\frac{3e^{-6x}dx}{2e^{-10x}}$$

$$k = \frac{3}{2}\int e^{-6x}\cdot e^{10x}dx$$

$$k = \frac{3}{2}\int e^{4x}dx$$

$$k = \frac{3}{2}\cdot\frac{1}{4}e^{4x} + C$$

$$k = \frac{3}{8}e^{4x} + C$$

$$\left(\int e^{4x}dx = \left|\begin{matrix} t = 4x \\ dt = 4dx \\ dx = \frac{dt}{4}\end{matrix}\right| = \int e^{t}\cdot\frac{dt}{4} = \frac{1}{4}e^{t} = \frac{1}{4}e^{4x} + C\right)$$

$$y = e^{-10x}\cdot\left(\frac{3}{8}e^{4x} + C\right)$$

$$y = \frac{\frac{3}{8}e^{4x} + C}{e^{10x}}$$

$$y = \frac{3}{8e^{6x}} + \frac{C}{e^{10x}}$$

3) $\dfrac{y'}{e^{5x-6}} - y^2 = 0$

$$\frac{dy}{dx\, e^{5x-6}} = y^2$$

$$\int \frac{dy}{y^2} = \int e^{5x-6}\,dx$$

$$-\frac{1}{y} = \frac{1}{5}e^{5x-6} + C$$

$$\boxed{y = -\frac{5}{e^{5x-6}} + C}$$

$$\left(\int \frac{dy}{y^2} = \int y^{-2}dy = \frac{y^{-1}}{-1} = \frac{-1}{y} \right.$$

$$\left. \int e^{5x-6}dx = \left| \begin{array}{l} t = 5x-6 \\ dt = 5dx \\ dx = \frac{dt}{5} \end{array} \right| = \int e^t \cdot \frac{dt}{5} = \frac{1}{5}e^{5x-6} + C \right)$$

4) $y' = (y+1)\sin x$

$$\frac{dy}{dx} = y\sin x + \underbrace{\sin x}_{=0}$$

$$\frac{dy}{dx} = y \cdot \sin x$$

$$\int \frac{dy}{y} = \int \sin x\, dx$$

$$\ln|y| = -\cos x + C$$

$$\ln|y| = \ln(e^{-\cos x} \cdot K)$$

$$y = e^{-\cos x} \cdot K$$

$$\left(\int \frac{\sin x}{e^{-\cos x}}dx = \left| \begin{array}{l} t = -\cos x \\ dt = \sin x\, dx \\ dx = \frac{dt}{\sin x} \end{array} \right| = \int \frac{\cancel{\sin x}}{e^t} \cdot \frac{dt}{\cancel{\sin x}} = \right.$$

$$= \int e^{-t}dt = \left| \begin{array}{l} s = -t \\ ds = -dt \\ dt = -ds \end{array} \right| = \int e^s(-ds) = -e^s = -e^{-t} =$$

$$\left. = -e^{-(-\cos x)} = -e^{\cos x} + C \right)$$

$$y = e^{-\cos x} \cdot k(x)$$

$$y' = e^{-\cos x} \cdot \sin x \cdot k(x) + e^{-\cos x} k'(x)$$

$$e^{-\cos x} \cdot \sin x \cdot k(x) + e^{-\cos x} k'(x) = (e^{-\cos x} \cdot k(x) + 1)\sin x$$

$$\cancel{e^{-\cos x} \sin x \cdot k(x)} + e^{-\cos x} \cdot k'(x) = \cancel{e^{-\cos x} \sin x \cdot k(x)} + \sin x$$

$$e^{-\cos x}\frac{dk}{dx} = \sin x$$

$$\int dk = \int \frac{\sin x}{e^{-\cos x}}dx$$

$$k = -e^{\cos x} + C$$

$$\boxed{\begin{array}{c} y = e^{-\cos x}(-e^{\cos x} + C) \\ y = \dfrac{-e^{\cos x} + C}{e^{\cos x}} \\ y = -1 + \dfrac{C}{e^{\cos x}} \end{array}}$$

5) $y' + 2y = e^{2x}$

$$y' + 2y = 0$$

$$\frac{dy}{dx} + 2y = 0$$

$$\frac{dy}{dx} = -2y$$

$$\int \frac{dy}{y} = -2\int dx$$

$$\ln|y| = -2x + C$$

$$\ln|y| = \ln e^{-2x} + \ln K$$

$$\cancel{\ln}|y| = \cancel{\ln}(e^{-2x} \cdot K)$$

$$y = e^{-2x} \cdot K$$

$$y = e^{-2x} \cdot k(x)$$

$$y' = e^{-2x}(-2)k(x) + e^{-2x}k'(x)$$

$$\cancel{-2e^{-2x}k(x)} + e^{-2x}k'(x) + \cancel{2e^{-2x}k(x)} = e^{2x}$$

$$e^{-2x}\frac{dk}{dx} = e^{2x}$$

$$\int dk = \int \frac{e^{2x}}{e^{-2x}}\,dx$$

$$k = \int e^{2x} \cdot e^{2x}\,dx$$

$$k = \int e^{4x}\,dx$$

$$k = \frac{e^{4x}}{4} + C$$

$$\left(\int e^{4x}\,dx = \left| \begin{array}{l} t = 4x \\ dt = 4\,dx \\ dx = \frac{dt}{4} \end{array} \right| = \int e^{t} \cdot \frac{dt}{4} = \frac{e^{t}}{4} = \frac{e^{4x}}{4} + C \right)$$

$$\boxed{\begin{array}{l} y = e^{-2x} \cdot \left(\frac{e^{4x}}{4} + C\right) \\ y = \frac{\frac{e^{4x}}{4} + C}{e^{2x}} \\ y = \frac{e^{2x}}{4} + \frac{C}{e^{2x}} \end{array}}$$

6) $y' - y \cdot \operatorname{cotg} x = \sin^2 x$

$$\frac{dy}{dx} = y \cdot \operatorname{cotg} x$$

$$\int \frac{dy}{y} = \int \operatorname{cotg} x \cdot dx$$

$$\ln|y| = \ln|\sin x| + C$$

$$\ln|y| = \ln|\sin x \cdot K|$$

$$y = \sin x \cdot K$$

$$\left(\int \operatorname{cotg} x \, dx = \int \frac{\cos x}{\sin x} dx = \left| \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int \frac{\cancel{\cos x}}{t} \cdot \frac{dt}{\cancel{\cos x}} = \ln|t| = \ln|\sin x| + C \right)$$

$$y = \sin x \cdot k(x)$$

$$y' = \cos x \cdot k(x) + \sin x \cdot k'(x)$$

$$\cos x \cdot k(x) + \sin x \cdot k'(x) - \sin x \cdot k(x) \cdot \operatorname{cotg} x = \sin^2 x$$

$$\cos x \, k(x) + \sin x \cdot k'(x) - \cancel{\sin x} \, k(x) \cdot \frac{\cos x}{\cancel{\sin x}} = \sin^2 x$$

$$\cancel{\cos x \, k(x)} + \sin x \cdot k'(x) - \cancel{\cos x \, k(x)} = \sin^2 x$$

$$\sin x \cdot \frac{dk}{dx} = \sin^2 x$$

$$dk = \frac{\sin^{\cancel{2}} x \, dx}{\cancel{\sin x}}$$

$$\int dk = \int \sin x \, dx$$

$$k = -\cos x + C$$

$$\boxed{\begin{array}{l} y = \sin x \cdot (-\cos x + C) \\ y = -\sin x \cdot \cos x + \sin x \cdot C \end{array}}$$

7) $y' \cdot \sin^2(2x-5) - y^2 = 0$

$$\frac{dy}{dx} \cdot \sin^2(2x-5) = y^2$$

$$\int \frac{dy}{y^2} = \int \frac{dx}{\sin^2(2x-5)}$$

$$-\frac{1}{y} = -\frac{1}{2} \operatorname{cotg}(2x-5) + C$$

$$y = 2 \cdot \frac{1}{\operatorname{cotg}(2x-5)} + C$$

$$y = 2 \operatorname{tg}(2x-5) + C$$

$$\left(\int \frac{dy}{y^2} = \int y^{-2} dy = \frac{y^{-1}}{-1} = \frac{-1}{y} \right.$$

$$\int \frac{dx}{\sin^2(2x-5)} = \left| \begin{array}{l} t = 2x-5 \\ dt = 2 \, dx \\ dx = \frac{dt}{2} \end{array} \right| = \int \frac{\frac{dt}{2}}{\sin^2 t} = \frac{1}{2} \int \frac{dt}{\sin^2 t} =$$

$$\left. = \frac{1}{2}(-\operatorname{cotg} t) = -\frac{1}{2} \operatorname{cotg}(2x-5) \right)$$

8) $3y' + 3y = 7x - 2$

$$3\frac{dy}{dx} + 3y = 0$$

$$\cancel{3}\frac{dy}{dx} = -\cancel{3}y$$

$$\frac{dy}{dx} = -y$$

$$\int\frac{dy}{y} = -\int dx$$

$$\ln|y| = -x + C$$

$$\ln|y| = \ln e^{-x} + \ln K$$

$$\ln|y| = \ln(e^{-x}\cdot K)$$

$$y = e^{-x}\cdot K$$

$$\int x\cdot e^{x} = \left| \begin{matrix} u' = e^{x} & v = x \\ u = e^{x} & v' = 1 \end{matrix} \right| =$$

$$= e^{x}\cdot x - \int e^{x}\cdot 1\,dx = e^{x}\cdot x - e^{x} + C$$

$$y = e^{-x}\cdot k(x)$$

$$y' = -e^{-x}\cdot k(x) + e^{-x}\cdot k'(x)$$

$$3\left(-e^{-x}\cdot k(x) + e^{-x}\cdot k'(x)\right) + 3e^{-x}\cdot k(x) = 7x - 2$$

$$-\cancel{3e^{-x}k(x)} + 3e^{-x}\cdot k'(x) + \cancel{3e^{-x}\cdot k(x)} = 7x - 2$$

$$3e^{-x}\cdot k'(x) = 7x - 2$$

$$3e^{-x}\frac{dk}{dx} = 7x - 2$$

$$\int dk = \int\frac{7x-2}{3e^{-x}}dx$$

$$\int dk = \int\frac{7x}{3e^{-x}}dx - \int\frac{2}{3e^{-x}}dx$$

$$k = \frac{7}{3}\int\frac{x}{e^{-x}}dx - \frac{2}{3}\int\frac{1}{e^{-x}}dx$$

$$k = \frac{7}{3}\int x\cdot e^{x}dx - \frac{2}{3}\int e^{x}$$

$$k = \frac{7}{3}\left(e^{x}\cdot x - e^{x}\right) - \frac{2}{3}e^{x} + C$$

$$k = \frac{7e^{x}\cdot x}{3} - \frac{7e^{x}}{3} - \frac{2e^{x}}{3} + C$$

$$k = \frac{7e^{x}\cdot x - 9e^{x}}{3} + C$$

$$\boxed{\begin{aligned} y &= e^{-x}\left(\frac{7e^{x}\cdot x - 9e^{x}}{3} + C\right) \\ y &= \frac{7e^{x}\cdot x - 9e^{x} + C}{3e^{x}} \\ y &= \frac{7x}{3} - 3 + \frac{C}{e^{x}} \end{aligned}}$$