

PAA

DÚ 11 - ŘEŠENÍ

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INVERZNÍ MATICE,

MATICOVÉ ROVNICE

$$\begin{aligned}
 1) \quad & x + 3y - 4z = 4 \\
 & 2x + 7y + 4z = 4 \\
 & x + 4y + 3z + 2z = 2 \\
 & \underline{2x + 7y + 5z + 2z = 2}
 \end{aligned}$$

$$\begin{pmatrix} 1 & 3 & 0 & -4 & | & 4 \\ 2 & 7 & 4 & 0 & | & 4 \\ 1 & 4 & 3 & 2 & | & 2 \\ 2 & 7 & 5 & 2 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 0 & -4 & | & 4 \\ 0 & 1 & 4 & 8 & | & -4 \\ 0 & -1 & -3 & -6 & | & 2 \\ 0 & 1 & 5 & 10 & | & -6 \end{pmatrix} \begin{matrix} -2I+II \\ I-III \\ -II+IV \end{matrix} \sim \begin{pmatrix} 1 & 3 & 0 & -4 & | & 4 \\ 0 & 1 & 4 & 8 & | & -4 \\ 0 & 0 & 1 & 2 & | & -2 \\ 0 & 0 & -1 & -2 & | & 2 \end{pmatrix} \begin{matrix} \\ \\ II+III \\ II-IV \end{matrix}$$

$$\begin{aligned}
 \underline{z = a} \\
 1z + 2a = -2 \\
 \underline{z = -2a - 2}
 \end{aligned}$$

$$\begin{aligned}
 y + 4(-2a - 2) + 8a = -4 \\
 y - 8a - 8 + 8a = -4 \\
 \underline{y = 4}
 \end{aligned}$$

$$\begin{aligned}
 x + 3 \cdot 4 - 4a = 4 \\
 x + 12 - 4a = 4 \\
 \underline{x = 4a - 8}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & x - 2y + z = 2 \\
 & 2x - 3y - z + 9z = -1 \\
 & x - 5z + 9z = 1 \\
 & \underline{-2x + 3y + 4z - 3z = -8}
 \end{aligned}$$

$$\begin{pmatrix} 1 & -2 & 0 & 1 & | & 2 \\ 2 & -3 & -1 & 9 & | & -1 \\ 1 & 0 & -5 & 9 & | & 1 \\ -2 & 3 & 4 & -3 & | & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 & 1 & | & 2 \\ 0 & 1 & -1 & 7 & | & -5 \\ 0 & 2 & -5 & 8 & | & -1 \\ 0 & -1 & 4 & -1 & | & -4 \end{pmatrix} \begin{matrix} -2I+II \\ I-III \\ 2I+IV \end{matrix} \sim \begin{pmatrix} 1 & -2 & 0 & 1 & | & 2 \\ 0 & 1 & -1 & 7 & | & -5 \\ 0 & 0 & -3 & -6 & | & 9 \\ 0 & 0 & 3 & 6 & | & -9 \end{pmatrix} \begin{matrix} \\ \\ -2I+III \\ I+IV \end{matrix}$$

$$\begin{aligned}
 \underline{z = a} \\
 -3z - 6a = 9 \\
 -3z = 6a + 9 \\
 \underline{z = -2a - 3}
 \end{aligned}$$

$$\begin{aligned}
 y - 1(-2a - 3) + 7a = -5 \\
 y + 2a + 3 + 7a = -5 \\
 \underline{y = -9a - 8}
 \end{aligned}$$

$$\begin{aligned}
 x - 2(-9a - 8) + a = 2 \\
 x + 18a + 16 + a = 2 \\
 \underline{x = -19a - 14}
 \end{aligned}$$

$$3) \quad A = \begin{pmatrix} 1 & -1 & 1 & 1 \\ 5 & -3 & -3 & -5 \\ 5 & -3 & 1 & 4 \\ 3 & -2 & 3 & h \end{pmatrix}$$

$$\begin{pmatrix} -10 - 2(h-3) = -10 - 2h + 6 = -2h - 4 \\ 2 \cdot 9 + (-2h - 4) = 18 - 2h - 4 = -2h + 14 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -8 & -10 \\ 0 & 2 & -4 & -1 \\ 0 & 1 & 0 & h-3 \end{pmatrix} \begin{array}{l} -5I+II \\ -5I+III \\ -3I+IV \end{array} \sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -8 & -10 \\ 0 & 0 & 4 & 9 \\ 0 & 0 & -8 & -2h-4 \end{pmatrix} \begin{array}{l} -II+III \\ II-2IV \end{array} \sim \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 2 & -8 & -10 \\ 0 & 0 & 4 & 9 \\ 0 & 0 & 0 & -2h+14 \end{pmatrix} \begin{array}{l} 2III+IV \end{array}$$

$$\begin{aligned} -2h + 14 &= 0 \\ 2h &= 14 \\ \underline{h} &= 7 \end{aligned}$$

$$\begin{array}{l} / h=3 \dots \text{pro } h=7 \\ \backslash h=4 \dots \text{pro } h \neq 7 \end{array}$$

$$4) \quad A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & 3 & -2 & -7 \\ -3 & -5 & -2 & -1 \\ 2 & 1 & 5 & h \end{pmatrix}$$

$$\begin{pmatrix} -5 + 5(h-2) = -5 + 5h - 10 = 5h - 15 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 0 & -5 \\ 0 & -2 & 1 & 2 \\ 0 & -1 & 3 & h-2 \end{pmatrix} \begin{array}{l} 2I+II \\ 3I+III \\ -2I+IV \end{array} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 15 & 5h-15 \end{pmatrix} \begin{array}{l} 2II+5III \\ II+5IV \end{array} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 0 & -5 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5h-15 \end{pmatrix} \begin{array}{l} -3III+IV \end{array}$$

$$\begin{aligned} 5h - 15 &= 0 \\ 5h &= 15 \\ \underline{h} &= 3 \end{aligned}$$

$$\begin{array}{l} / h=3 \dots \text{pro } h=3 \\ \backslash h=4 \dots \text{pro } h \neq 3 \end{array}$$

$$5) \quad A = \begin{pmatrix} -2 & 1 & 1 \\ 3 & -3 & -2 \\ 3 & -1 & -1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -2 & 1 & 1 & 1 & 0 & 0 \\ 3 & -3 & -2 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} -2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & 3 & 2 & 0 \\ 0 & 1 & 1 & 3 & 0 & 2 \end{array} \right) \begin{array}{l} 3I+2II \\ 3I+2III \end{array} \sim \left(\begin{array}{ccc|ccc} -2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & 3 & 2 & 0 \\ 0 & 0 & 2 & 12 & 2 & 6 \end{array} \right) \begin{array}{l} \\ \\ II+3III \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} -2 & 1 & 0 & -5 & -1 & -3 \\ 0 & -3 & 0 & 9 & 3 & 3 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array} \right) \begin{array}{l} I-III(mult) \\ II+III(mult) \\ :2 \end{array} \sim \left(\begin{array}{ccc|ccc} -2 & 0 & 0 & -2 & 0 & -2 \\ 0 & 1 & 0 & -3 & -1 & -1 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array} \right) \begin{array}{l} I-II(mult) \\ :(-3) \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -3 & -1 & -1 \\ 0 & 0 & 1 & 6 & 1 & 3 \end{array} \right) \begin{array}{l} \\ \\ :(-2) \end{array}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -3 & -1 & -1 \\ 6 & 1 & 3 \end{pmatrix}$$

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$$A \cdot A^{-1} = \begin{pmatrix} -2 & 1 & 1 \\ 3 & -3 & -2 \\ 3 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ -3 & -1 & -1 \\ 6 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 \cdot 1 + 1 \cdot (-3) + 1 \cdot 6 & -2 \cdot 0 + 1 \cdot (-1) + 1 \cdot 1 & -2 \cdot 1 + 1 \cdot (-1) + 1 \cdot 3 \\ 3 \cdot 1 - 3 \cdot (-3) - 2 \cdot 6 & 3 \cdot 0 - 3 \cdot (-1) - 2 \cdot 1 & 3 \cdot 1 - 3 \cdot (-1) - 2 \cdot 3 \\ 3 \cdot 1 - 1 \cdot (-3) - 1 \cdot 6 & 3 \cdot 0 - 1 \cdot (-1) - 1 \cdot 1 & 3 \cdot 1 - 1 \cdot (-1) - 1 \cdot 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2 - 3 + 6 & 0 - 1 + 1 & -2 - 1 + 3 \\ 3 + 9 - 12 & 0 + 3 - 2 & 3 + 3 - 6 \\ 3 + 3 - 6 & 0 + 1 - 1 & 3 + 1 - 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$A^{-1} \cdot A = \begin{pmatrix} 1 & 0 & 1 \\ -3 & -1 & -1 \\ 6 & 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 & 1 \\ 3 & -3 & -2 \\ 3 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -2+3 & 1-1 & 1-1 \\ 6-3-3 & -3+3+1 & -3+2+1 \\ -12+3+9 & 6-3-3 & 6-2-3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$6) A \cdot X = B$$

$$A = \begin{pmatrix} 1 & 1 & -3 \\ 2 & 1 & -4 \\ 1 & 1 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -2 & 3 \\ -9 & 5 \\ -2 & 3 \end{pmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -3 & 1 & 0 & 0 \\ 2 & 1 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right) \begin{array}{l} -2I+II \\ -I+III \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -2 & 0 & 3 \\ 0 & -1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right) \begin{array}{l} I+3III \\ II+III \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 1 & 4 \\ 0 & -1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right) \begin{array}{l} I+II \\ \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 1 & 4 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} -5 & 1 & 4 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} -5 & 1 & 4 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 \\ -9 & 5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 10-9-8 & -15+5+12 \\ -6+9+2 & 9-5-3 \\ 2-2 & -3+3 \end{pmatrix} = \begin{pmatrix} -7 & 2 \\ 5 & 1 \\ 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -7 & 2 \\ 5 & 1 \end{pmatrix}$$

$$7) A \cdot X + B = 3A$$

$$A = \begin{pmatrix} -2 & 1 \\ 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} -4 & 0 \\ -1 & 9 \end{pmatrix}$$

$$A \cdot X + B = 3A$$

$$A \cdot X = 3A - B$$

$$X = A^{-1}(3A - B)$$

$$3A = 3 \cdot \begin{pmatrix} -2 & 1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 3 & 12 \end{pmatrix}$$

$$3A - B = \begin{pmatrix} -6 & 3 \\ 3 & 12 \end{pmatrix} - \begin{pmatrix} -4 & 0 \\ -1 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} -2 & 1 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{cc|cc} -2 & 1 & 1 & 0 \\ 0 & 9 & 1 & 2 \end{array} \right) \begin{array}{l} I+2II \\ I+2II \end{array} \sim \left(\begin{array}{cc|cc} 18 & 0 & -8 & 2 \\ 0 & 9 & 1 & 2 \end{array} \right) \begin{array}{l} -9I+II \\ -9I+II \end{array} \sim \left(\begin{array}{cc|cc} 1 & 0 & -\frac{4}{9} & \frac{1}{9} \\ 0 & 1 & \frac{1}{9} & \frac{2}{9} \end{array} \right) \begin{array}{l} :18 \\ :9 \end{array}$$

$$A^{-1} = \begin{pmatrix} -\frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix}$$

$$X = A^{-1}(3A - B) = \begin{pmatrix} -\frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -\frac{4}{9} + \frac{4}{9} & -\frac{12}{9} + \frac{3}{9} \\ \frac{4}{9} + \frac{8}{9} & \frac{3}{9} + \frac{6}{9} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{9}{9} \\ \frac{9}{9} & \frac{9}{9} \end{pmatrix} =$$

$$\boxed{\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}}$$

$$8) X \cdot A = B$$

$$A = \begin{pmatrix} 1 & -6 & -2 \\ 1 & 1 & -1 \\ 1 & -4 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 4 & 1 \\ -2 & -9 & 0 \end{pmatrix}$$

$$X \cdot A = B$$

$$X = B \cdot A^{-1}$$

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 1 & -6 & -2 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & -4 & -2 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & -6 & -2 & 1 & 0 & 0 \\ 0 & -7 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{array} \right) \begin{array}{l} I-II \\ I-III \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & -6 & -2 & 1 & 0 & 0 \\ 0 & -7 & -1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 8 & -1 & -7 \end{array} \right) \begin{array}{l} \\ \\ II+7I \end{array} \\ & \sim \left(\begin{array}{ccc|ccc} 1 & -6 & 0 & -15 & 2 & 14 \\ 0 & -7 & 0 & -7 & 0 & 7 \\ 0 & 0 & -1 & 8 & -1 & -7 \end{array} \right) \begin{array}{l} I-2III \\ II-III \end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -9 & 2 & 8 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -8 & 1 & 7 \end{array} \right) \begin{array}{l} I+6II(\text{wrong}) \\ I:(-7) \\ I:(-1) \end{array} \end{aligned}$$

$$A^{-1} = \begin{pmatrix} -9 & 2 & 8 \\ 1 & 0 & -1 \\ -8 & 1 & 7 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 4 & 1 \\ -2 & -9 & 0 \end{pmatrix} \cdot \begin{pmatrix} -9 & 2 & 8 \\ 1 & 0 & -1 \\ -8 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 4-8 & 1 & -4+7 \\ 18-9 & -4 & -16+9 \end{pmatrix} = \begin{pmatrix} -4 & 1 & 3 \\ 9 & -4 & -7 \end{pmatrix}$$