DÚ 2 - ŘEŠENÍ

1) $\log \frac{x^2-6x+5}{-2x^2-10x-12} - 2\sqrt{81-x^2}$

$-2x^2-10x-12 \neq 0 \quad /:(-2)$ $\wedge$ $\frac{x^2-6x+5}{-2x^2-10x-12} > 0$ $\wedge$ $81-x^2 \geq 0$

$x^2+5x+6 \neq 0$

$(x+2)(x+3) \neq 0$

$x \neq -2 \quad x \neq -3$

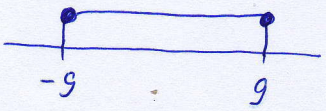
N.B.: $x^2-6x+5=0$

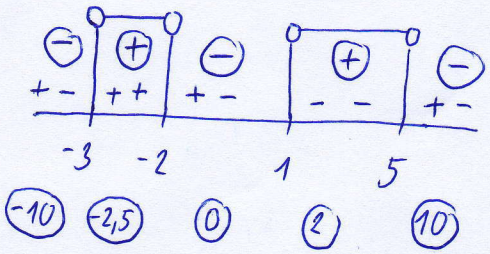
$(x-5)(x-1)=0$

$5 \quad 1$

$81 \geq x^2$

$9 \geq |x|$

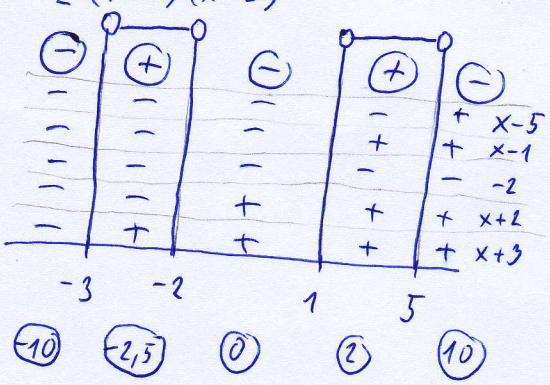




DALŠÍ MOŽNOSTI ŘEŠENÍ 2. PODMÍNKY:

$\frac{x^2-6x+5}{-2x^2-10x-12} > 0$

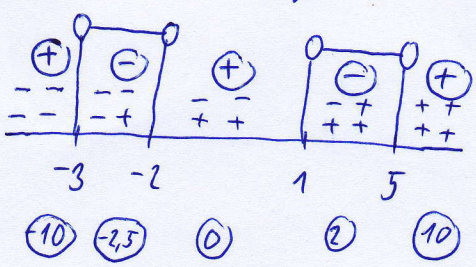
$\frac{(x-5)(x-1)}{-2(x+2)(x+3)} > 0$



nebo $\frac{x^2-6x+5}{-2x^2-10x-12} > 0 \quad /:(-2)$

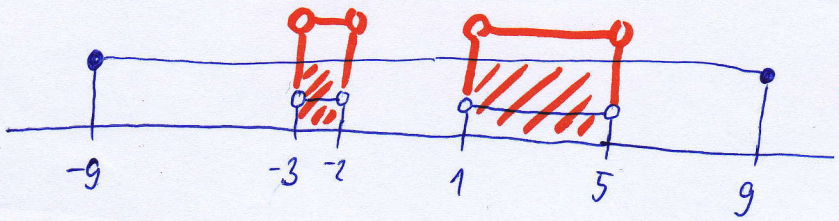
$\frac{x^2-6x+5}{x^2+5x+6} < 0$

$\frac{(x-5)(x-1)}{(x+2)(x+3)} < 0$



Jde o to, že :(-2) si můžeme beztrestně dovolit u rovnice.

U nerovnice musíme buď otočit znaménko, nebo to vůbec nedělit, ale jenom vytknout.



$x \in (-3;-2) \cup (1;5)$

2) $\sqrt{\frac{x^2+2x-8}{10^x-100}} + 3\log_2(9-x^2)$

$10^x - 100 \neq 0 \quad \wedge \quad \frac{x^2+2x-8}{10^x-100} \geq 0 \quad \wedge \quad 9-x^2 > 0$

$10^x \neq 100$

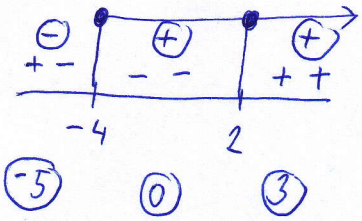
$10^x \neq 10^2$

$x \neq 2$

M.B.: $x^2+2x-8 = 0$

$(x-2)(x+4) = 0$

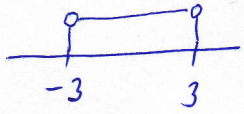
$2 \quad -4$

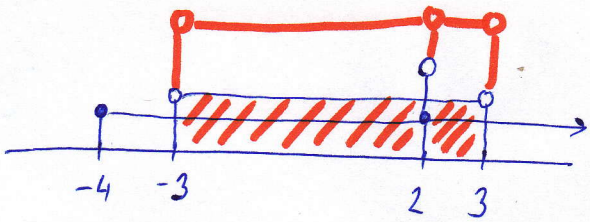


$-5 \quad 0 \quad 3$

$9 > x^2$

$3 > |x|$





$D(f) = (-3;2) \cup (2;3) = (-3;3) \setminus \{2\}$

3) $\sqrt{\frac{2x^2+6x+4}{x^2-2x+3}} + 6\log_7(1-x^2)$

$x^2-2x+3 \neq 0 \quad \wedge \quad \frac{2x^2+6x+4}{x^2-2x+3} \geq 0 \quad \wedge \quad 1-x^2 > 0$

$D = 4-12 = -8$

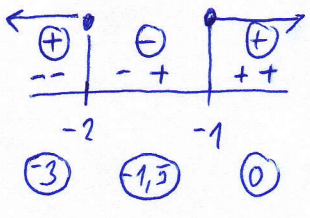
nemá řešení

$\frac{2x^2+6x+4}{x^2-2x+3}$ ⊕

$2x^2+6x+4 \geq 0 \quad / : 2$

$x^2+3x+2 \geq 0$

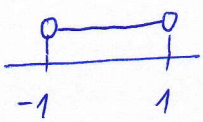
$(x+1)(x+2) \geq 0$

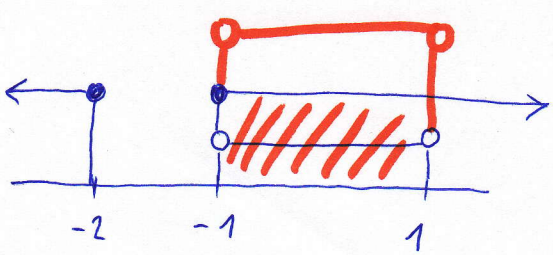


$-3 \quad -1{,}5 \quad 0$

$1 > x^2$

$1 > |x|$





$D(f) = (-1;1)$

4) $\sqrt{\frac{x^2+10x+21}{-x^2+10x-9}} + 2\log_7(64-x^2)$

$-x^2+10x-9 \neq 0 \quad \wedge \quad \frac{x^2+10x+21}{-x^2+10x-9} \geq 0 \quad \wedge \quad 64-x^2 > 0$

$x^2-10x+9 \neq 0$

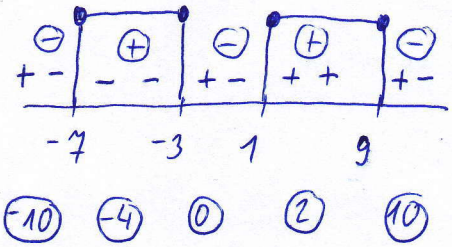
$(x-1)(x-9) \neq 0$

$x \neq 1 \quad x \neq 9$

N.B.: $x^2+10x+21 = 0$

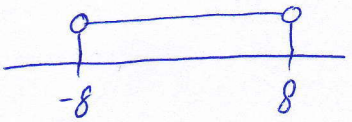
$(x+3)(x+7) = 0$

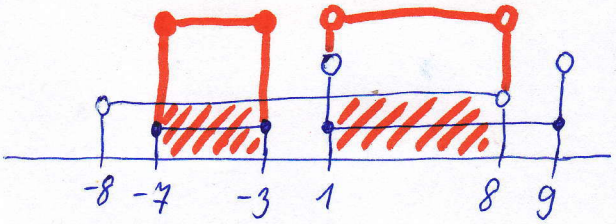
$-3 \quad -7$



$x^2 < 64$

$|x| < 8$





$D(f) = \langle -7; -3 \rangle \cup (1; 8)$

5) $\log_5 \frac{x^2-2x-8}{64-2^x} - 2\sqrt{16-x^2}$

$64 \neq 2^x \quad \wedge \quad \frac{x^2-2x-8}{64-2^x} > 0 \quad \wedge \quad 16-x^2 \geq 0$

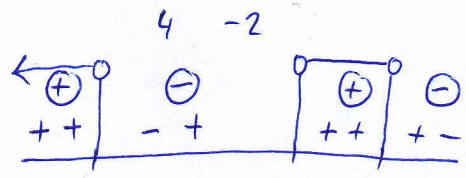
$2^x \neq 2^6$

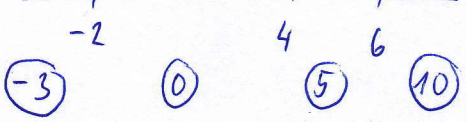
$x \neq 6$

N.B.: $x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

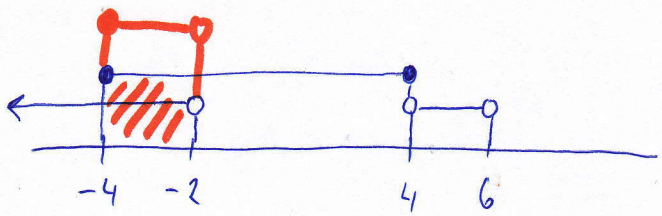
$4 \quad -2$





$x^2 \leq 16$

$|x| \leq 4$



$D(f) = \langle -4; -2)$

6) $\sqrt{\frac{2x^2+6x+4}{x^2-2x+3}} + 6\log_4(1-x^2)$

$x^2-2x+3 \neq 0 \quad \wedge \quad \frac{2x^2+6x+4}{x^2-2x+3} \geq 0 \quad \wedge \quad 1-x^2 > 0$

$D = 4-12 = -8$

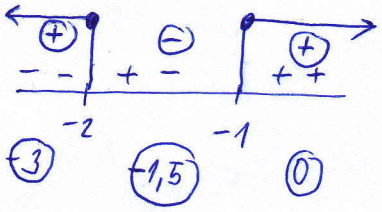
nemá reálne riešenie

$\frac{2x^2+6x+4}{x^2-2x+3}$ je vždy $\oplus$

$2x^2+6x+4 \geq 0 \quad /:2$

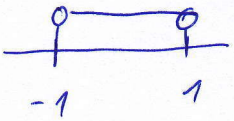
$x^2+3x+2 \geq 0$

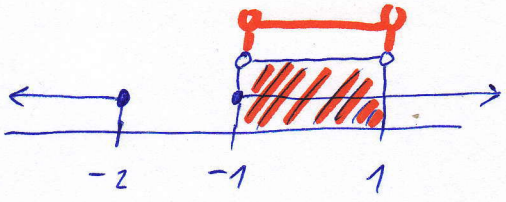
$(x+2)(x+1) \geq 0$



$1 > x^2$

$1 > |x|$





$D(f) = (-1;1)$

7) $\log_5 \frac{-x-7}{-2x^2+4x-2} + 4\sqrt{x^2-25}$

$-2x^2+4x-2 \neq 0 \quad \wedge \quad \frac{-x-7}{-2x^2+4x-2} > 0 \quad \wedge \quad x^2-25 \geq 0$

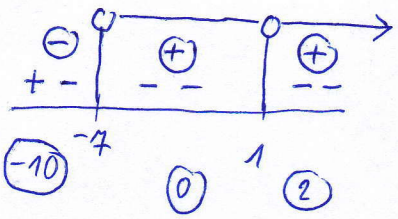
$x^2-2x+1 \neq 0$

$(x-1)(x-1) \neq 0$

$x \neq 1$

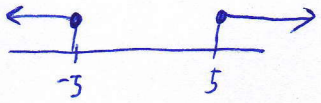
N.B.: $-x-7 = 0$

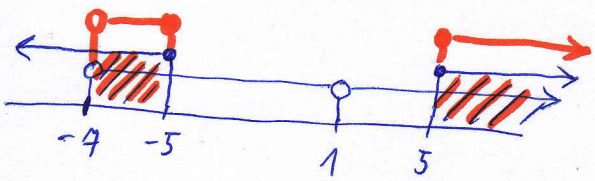
$-7 = x$



$x^2 \geq 25$

$|x| \geq 5$





$D(f) = (-7;-5\rangle \cup \langle 5;\infty)$

8) $\log_3 \frac{-x^2+6x+16}{x-1} - 5\sqrt{3-\log_3(x+6)}$

$x-1 \neq 0 \quad \wedge \quad \frac{-x^2+6x+16}{x-1} > 0 \quad \wedge \quad x+6 \geq 0 \quad \wedge \quad 3-\log_3(x+6) \geq 0$

$x \neq 1$

N.B.: $x^2-6x-16=0$

$(x+2)(x-8)=0$

$-2 \quad 8$

$x > -6$

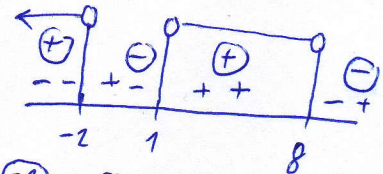
$3 \geq \log_3(x+6)$

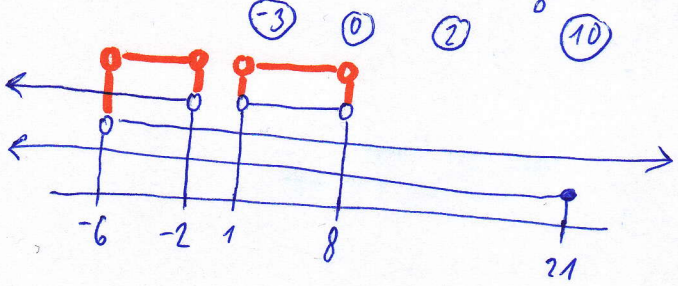
$\log_3 3^3 \geq \log_3(x+6)$

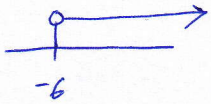
$3^3 \geq x+6$

$27 \geq x+6$

$21 \geq x$







$D(f) = (-6;-2) \cup (1;8)$

9) $\sqrt{\frac{-x^2+4x+5}{x-4}} + 7\log(1-\log(x+2))$

$x-4 \neq 0 \quad \wedge \quad \frac{-x^2+4x+5}{x-4} \geq 0 \quad \wedge \quad x+2 > 0 \quad \wedge \quad 1-\log(x+2) > 0$

$x \neq 4$

N.B.: $x^2-4x-5=0$

$(x-5)(x+1)=0$

$5 \quad -1$

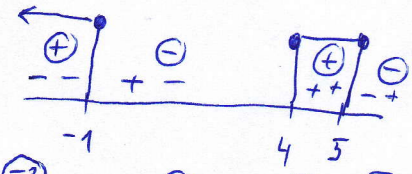
$x > -2$

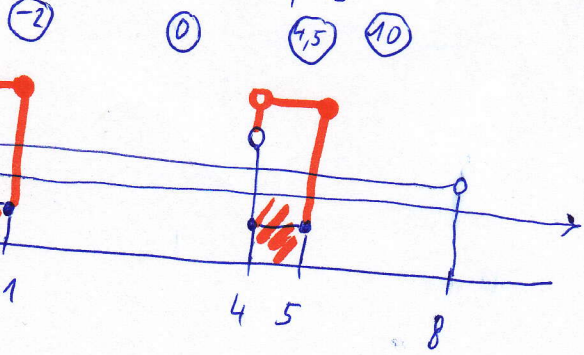
$1 > \log(x+2)$

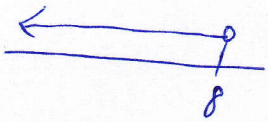
$\log 10^1 > \log(x+2)$

$10 > x+2$

$8 > x$







$D(f) = (-2;-1\rangle \cup (4;5\rangle$

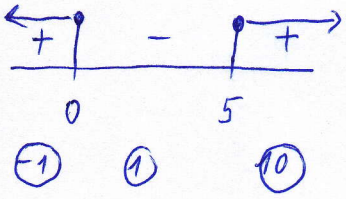
10) $\sqrt{x^2-5x} - 3 \arccos \frac{x}{x-3}$

$x^2-5x \geq 0 \quad \wedge \quad x-3 \neq 0 \quad \wedge \quad \frac{x}{x-3} \geq -1 \quad \wedge \quad \frac{x}{x-3} \leq 1$

$x \neq 3$

$x(x-5) \geq 0$

N.B.: 0; 5



(-1) (1) (10)

$\frac{x}{x-3} + 1 \geq 0$

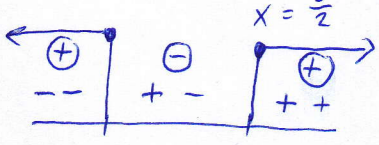
$\frac{x+x-3}{x-3} \geq 0$

$\frac{2x-3}{x-3} \geq 0$

N.B.: 3; $2x-3=0$

$2x=3$

$x=\frac{3}{2}$



(0) $\frac{3}{2}$ (2) 3 (4)

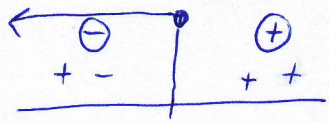
$\frac{x}{x-3} - 1 \leq 0$

$\frac{x-(x-3)}{x-3} \leq 0$

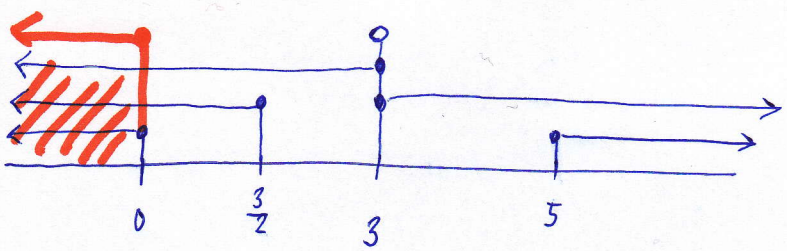
$\frac{x-x+3}{x-3} \leq 0$

$\frac{3}{x-3} \leq 0$

N.B.: 3



(0) 3 (4)



$D(f) = (-\infty; 0\rangle$