PaE, Info, Syl

DÚ 4 - ŘEŠENÍ

Taylorův polynom

1) $f(x) = x + \ln(x^2 - x + 1)$

$a = 1$

$$f'(x) = 1 + \frac{1}{x^2-x+1} \cdot (2x-1) = 1 + \frac{2x-1}{x^2-x+1}$$

$$f''(x) = 0 + \frac{2(x^2-x+1)-(2x-1)(2x-1)}{(x^2-x+1)^2} = \frac{2x^2-2x+2-(4x^2-4x+1)}{(x^2-x+1)^2} = \frac{-2x^2+2x+1}{(x^2-x+1)^2}$$

$$f'''(x) = \frac{(-4x+2)\cdot(x^2-x+1)-(-2x^2+2x+1)\cdot 2(x^2-x+1)^1\cdot(2x-1)}{(x^2-x+1)^4}$$

$$f(a) = 1 + \ln(1^2-1+1) = 1 + \ln 1 = 1 + 0 = 1$$

$$f'(a) = 1 + \frac{2\cdot 1 - 1}{1^2-1+1} = 1 + \frac{2-1}{1} = 1 + \frac{1}{1} = 1 + 1 = 2$$

$$f''(1) = \frac{-2\cdot 1 + 2 + 1}{1^2} = \frac{-1}{1} = +1$$

$$f'''(1) = \frac{(-4+2)\cdot(1^2-1+1)-(-2+2+1)\cdot 2\cdot(1-1+1)\cdot(2-1)}{1^4} = \frac{-2\cdot 1 - 1\cdot 2\cdot 1\cdot 1}{1} = -4$$

$$T_3(1) = 1 + \frac{2}{1!}(x-1)^1 + \frac{1}{2!}(x-1)^2 + \frac{-4}{3!}(x-1)^3 =$$

$$= 1 + 2(x-1) + \frac{1}{2}(x-1)^2 - \frac{2}{3}(x-1)^3$$

2) $f(x) = \cos(\pi - x) + \sin(-2x) + \frac{\sqrt{2}}{2}$

$a = \frac{\pi}{4}$

$f'(x) = -\sin(\pi - x)\cdot(-1) + \cos(-2x)\cdot(-2) = \underline{\sin(\pi - x) - 2\cos(-2x)}$

$f''(x) = +\cos(\pi - x)\cdot(-1) - 2(\sin(-2x))\cdot(-2) = \underline{-\cos(\pi - x) - 4\sin(-2x)}$

$f'''(x) = \sin(\pi - x)\cdot(-1) - 4\cdot\cos(-2x)\cdot(-2) = \underline{-\sin(\pi - x) + 8\cos(-2x)}$

$\boxed{\pi - x}$: $\pi - \frac{\pi}{4} = \frac{4\pi - \pi}{4} = \frac{3\pi}{4} \rightarrow \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\rightarrow \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\boxed{-2x}$: $-\cancel{2}^{1}\cdot\frac{\pi}{\cancel{4}_{2}} = -\frac{\pi}{2} \rightarrow \sin\left(-\frac{\pi}{2}\right) = -1$

$\rightarrow \cos\left(-\frac{\pi}{2}\right) = 0$

$f\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + (-1) + \frac{\sqrt{2}}{2} = \underline{\underline{-1}}$

$f'\left(\frac{\pi}{4}\right) = +\frac{\sqrt{2}}{2} - 2\cdot 0 = \underline{\underline{\frac{\sqrt{2}}{2}}}$

$f''\left(\frac{\pi}{4}\right) = -\left(-\frac{\sqrt{2}}{2}\right) - 4\cdot(-1) = \frac{\sqrt{2}}{2} + 4$

$\cancel{f'''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + 8\cdot\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} - 4\sqrt{2} = \frac{-\sqrt{2} - 8\sqrt{2}}{2} = -\frac{9\sqrt{2}}{2}}$

$f'''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + 8\cdot 0 = -\frac{\sqrt{2}}{2}$

$T_3\left(\frac{\pi}{4}\right) = -1 + \frac{\frac{\sqrt{2}}{2}}{1!}\left(x - \frac{\pi}{4}\right)^1 + \frac{\frac{\sqrt{2}}{2} + 4}{2!}\cdot\left(x - \frac{\pi}{4}\right)^2 + \frac{-\frac{\sqrt{2}}{2}}{3!}\left(x - \frac{\pi}{4}\right)^3$

$\underline{\underline{T_3\left(\frac{\pi}{4}\right) = -1 + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \left(\frac{\sqrt{2}}{4} + 2\right)\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3}}$

3) $f(x) = x + \sqrt[5]{5x-34} - \sqrt[5]{5}$

$a = 7$

$$f(x) = x + (5x-34)^{\frac{1}{5}} - \sqrt[5]{5}$$

$$f'(x) = 1 + \frac{1}{5}\cdot(5x-34)^{-\frac{4}{5}}\cdot 5 = 1 + (5x-34)^{-\frac{4}{5}}$$

$$f''(x) = -\frac{4}{5}(5x-34)^{-\frac{9}{5}}\cdot 5 = -4(5x-34)^{-\frac{9}{5}}$$

$$f'''(x) = -4\cdot\left(-\frac{9}{5}\right)(5x-34)^{-\frac{14}{5}}\cdot 5 = 36\cdot(5x-34)^{-\frac{14}{5}}$$

$$f(7) = 7 + (5\cdot 7-34)^{\frac{1}{5}} - \sqrt[5]{5} = 7 + 1^{\frac{1}{5}} - \sqrt[5]{5} = \underline{\underline{8 - \sqrt[5]{5}}}$$

$$f'(7) = 1 + 1^{-\frac{4}{5}} = \underline{\underline{2}}$$

$$f''(7) = -4\cdot 1^{-\frac{9}{5}} = \underline{\underline{-4}}$$

$$f'''(7) = 36\cdot 1^{-\frac{14}{5}} = \underline{\underline{36}}$$

$$T_3(7) = 8 - \sqrt[5]{5} + \frac{2}{1!}(x-7)^1 + \frac{-4}{2!}(x-7)^2 + \frac{36}{3!}(x-7)^3 =$$

$$= \underline{\underline{8 - \sqrt[5]{5} + 2(x-7) - 2(x-7)^2 + 6(x-7)^3}}$$

4) $f(x) = \frac{x^2-7x+2}{e^x}$

$a = 0$

$$f'(x) = \frac{(2x-7)e^x - (x^2-7x+2)e^x}{(e^x)^2} = \frac{\cancel{e^x}[(2x-7)-(x^2-7x+2)]}{(e^x)^{\cancel{2}}} = \frac{-x^2+9x-9}{e^x}$$

$$f''(x) = \frac{(-2x+9)e^x - (-x^2+9x-9)e^x}{(e^x)^2} = \frac{\cancel{e^x}[(-2x+9)-(-x^2+9x-9)]}{(e^x)^{\cancel{2}}} = \frac{x^2-11x+18}{e^x}$$

$$f'''(x) = \frac{(2x-11)e^x - (x^2-11x+18)e^x}{(e^x)^2} = \frac{\cancel{e^x}[(2x-11)-(x^2-11x+18)]}{(e^x)^{\cancel{2}}} = \frac{-x^2+13x-29}{e^x}$$

$$f(0) = \frac{0^2-7\cdot 0+2}{e^0} = \frac{2}{1} = \underline{\underline{2}}$$

$$f'(0) = \frac{-0^2+9\cdot 0-9}{e^0} = -\frac{9}{1} = \underline{\underline{-9}}$$

$$f''(0) = \frac{0^2-11\cdot 0+18}{e^0} = \frac{18}{1} = \underline{\underline{18}}$$

$$f'''(0) = \frac{-0^2+13\cdot 0-29}{e^0} = -\frac{29}{1} = \underline{\underline{-29}}$$

$$T_3(0) = 2 + \frac{-9}{1!}(x-0)^1 + \frac{18}{2!}(x-0)^2 + \frac{-29}{3!}(x-0)^3 =$$

$$= \underline{\underline{2 - 9x + 9x^2 - \frac{29}{6}x^3}}$$