PAA

DÚ 4 - ŘEŠENÍ

LIMITY, ASYMPTOTY

1)
$$\lim_{x\to -2} \frac{\sqrt{4x+12}-2}{3x+5+e^{-2x-4}} = \frac{\sqrt{4\cdot(-2)+12}-2}{3\cdot(-2)+5+e^{0}} = \frac{\sqrt{4}-2}{-6+5+1} = \text{„}\frac{0}{0}\text{"}$$

L.P.:
$$\lim_{x\to -2} \frac{\frac{1}{\cancel{2}\sqrt{4x+12}}\cdot \cancel{4}^{2}-0}{3+0+e^{-2x-4}\cdot(-2)} = \frac{\frac{2}{\sqrt{-8+12}}}{3+e^{0}\cdot(-2)} = \frac{\frac{2}{2}}{3+1\cdot(-2)} = \frac{1}{3-2} = \frac{1}{1} = +1$$

2)
$$\lim_{x\to \frac{2}{3}\pi} \frac{3\cdot tg^2 x-9}{-\frac{1}{2}-\cos(5x)} = \frac{3\cdot tg^2(\frac{2}{3}\pi)-9}{-\frac{1}{2}-\cos(\frac{10}{3}\pi)} = \frac{3\cdot(-\sqrt{3})^2-9}{-\frac{1}{2}-(-\frac{1}{2})} = \frac{3\cdot 3-9}{-\frac{1}{2}+\frac{1}{2}} = \text{„}\frac{0}{0}\text{"}$$

$\cos\left(\frac{10}{3}\pi\right)$..... MOC VELKÝ ÚHEL

=> ODEČTU PERIODU 2π: $\cos\left(\frac{10}{3}\pi-2\pi\right)=\cos\left(\frac{10\pi-6\pi}{3}\right)=\cos\left(\frac{4}{3}\pi\right)$.... MOC VELKÝ ÚHEL

=> ZNOVU ODEČTU PERIODU: $\cos\left(\frac{4}{3}\pi-2\pi\right)=\cos\left(\frac{4\pi-6\pi}{3}\right)=\cos\left(-\frac{2\pi}{3}\right)$... MOC MALÝ ÚHEL

=> OD $\frac{4}{3}\pi$ ODEČTU JENOM PŮL PERIODY A VÍM, ŽE TOMU, CO VYJDE, MUSÍM ZMĚNIT ZNAMÍNKO:

$\cos\left(\frac{4}{3}\pi-\pi\right)=\cos\left(\frac{4\pi-3\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$ => $\cos\left(\frac{10}{3}\pi\right)=-\frac{1}{2}$

-> U $\sin\left(\frac{10}{3}\pi\right)$ POSTUJI OBDOBNĚ

L.P.:
$$\lim_{x\to \frac{2}{3}\pi} \frac{3\cdot 2\cdot tg\, x\cdot \frac{1}{\cos^2 x}-0}{0-[-\sin(5x)]\cdot 5} = \frac{6\cdot tg\left(\frac{2}{3}\pi\right)\cdot \frac{1}{\cos^2(\frac{2}{3}\pi)}}{+\sin\left(\frac{10}{3}\pi\right)\cdot 5} = \frac{6\cdot(-\sqrt{3})\cdot\frac{1}{(-\frac{1}{2})^2}}{-\frac{\sqrt{3}}{2}\cdot 5} =$$

$$= \frac{\frac{-6\sqrt{3}}{\frac{1}{4}}}{\frac{-5\sqrt{3}}{2}} = \frac{\frac{-6\sqrt{3}}{1}\cdot\frac{4}{1}}{\frac{-5\sqrt{3}}{2}} = \frac{-24\cancel{\sqrt{3}}}{1}\cdot\frac{2}{-5\cancel{\sqrt{3}}} = \frac{48}{5}$$

3) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{7-7\cot g^2 x}{\sin(3x)-\frac{\sqrt{2}}{2}} = \dfrac{7-7\cdot \cot g^2 \frac{\pi}{4}}{\sin\frac{3\pi}{4}-\frac{\sqrt{2}}{2}} = \dfrac{7-7\cdot 1^2}{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}} = „\dfrac{0}{0}"$

L.P.:

$$\lim_{x \to \frac{\pi}{4}} \frac{0-7\cdot 2\cdot \cot g\, x \cdot \frac{-1}{\sin^2 x}}{\cos(3x)\cdot 3 - 0} = \frac{-14\cdot \cot g \frac{\pi}{4}\cdot \frac{-1}{\sin^2\frac{\pi}{4}}}{\cos\frac{3\pi}{4}\cdot 3} = \frac{-14\cdot 1\cdot \frac{-1}{\left(\frac{\sqrt{2}}{2}\right)^2}}{-\frac{\sqrt{2}}{2}\cdot 3} =$$

$$= \frac{\frac{14}{\frac{2}{4}}}{\frac{-3\sqrt{2}}{2}} = \frac{\frac{14}{1}\cdot\frac{\cancel{4}^2}{\cancel{2}_1}}{\frac{-3\sqrt{2}}{2}} = \frac{28}{1}\cdot\frac{2}{-3\sqrt{2}} = -\frac{56}{3\sqrt{2}}$$

4) $\lim\limits_{x \to -4} \dfrac{-e^{5x+20}-(x+3)}{7-\sqrt{-2x+41}} = \dfrac{-e^{-20+20}-(-4+3)}{7-\sqrt{-2(-4)+41}} = \dfrac{-e^0-(-1)}{7-\sqrt{49}} = \dfrac{-1+1}{7-7} = „\dfrac{0}{0}"$

L.P.:

$$\lim_{x \to -4} \frac{-e^{5x+20}\cdot(5)-1}{0-\frac{1}{2\sqrt{-2x+41}}\cdot(-2)} = \frac{-e^0\cdot 5-1}{-\frac{-2}{2\sqrt{49}}} = \frac{-5-1}{\frac{2}{14}} = \frac{-6}{\frac{1}{7}} = \frac{-6}{1}\cdot\frac{7}{1} = -42$$

5) $f(x) = \dfrac{-2x^2-6x-1}{x-3}$ $\boxed{D(f) = \mathbb{R} \setminus \{3\}}$

$\lim\limits_{x \to 3^+} \dfrac{-2x^2-6x-1}{x-3} = \dfrac{-18-18-1}{0^+} = -\infty$ $\boxed{a_1 : x = 3}$

$k = \lim\limits_{x \to \pm\infty} \dfrac{-2x^2-6x-1}{x(x-3)} = \lim\limits_{x \to \pm\infty} \dfrac{-2x^2-6x-1}{x^2-3x} = \lim\limits_{x \to \pm\infty} \dfrac{-2x^2}{x^2} = -2$

$q = \lim\limits_{x \to \pm\infty} \left(\dfrac{-2x^2-6x-1}{x-3}-(-2x)\right) = \lim\limits_{x \to \pm\infty} \dfrac{-2x^2-6x-1+2x^2-6x}{x-3} = \lim\limits_{x \to \pm\infty} \dfrac{-12x-1}{x-3} =$

$= \lim\limits_{x \to \pm\infty} \dfrac{-12x}{x} = -12$ $\boxed{a_2 : y = -2x-12}$

$$\boxed{a : y = k\cdot x + q \qquad k = \lim_{x \to \pm\infty} \frac{f(x)}{x} \qquad q = \lim_{x \to \pm\infty} (f(x) - k\cdot x)}$$

6) $f(x) = \dfrac{2x^2 - 9x + 2}{x+1}$ $\boxed{D(f) = \mathbb{R} \setminus \{-1\}}$

$$\lim_{x \to -1^+} \frac{2x^2-9x+2}{x+1} = \frac{2 \cdot 1 - 9 \cdot (-1) + 2}{-1^+ + 1} = \frac{2+9+2}{0+} = \frac{13}{0+} = \underline{+\infty} \qquad \boxed{a_1 : x = -1}$$

$$k = \lim_{x \to \pm\infty} \frac{2x^2 - 9x + 2}{x^2 + x} = \lim_{x \to \pm\infty} \frac{2x^2}{x^2} = \underline{2}$$

$$q = \lim_{x \to \pm\infty} \left(\frac{2x^2 - 9x + 2}{x+1} - 2x \right) = \lim_{x \to \pm\infty} \frac{2x^2 - 9x + 2 - 2x^2 - 2x}{x+1} = \lim_{x \to \pm\infty} \frac{-11x + 2}{x+1} =$$

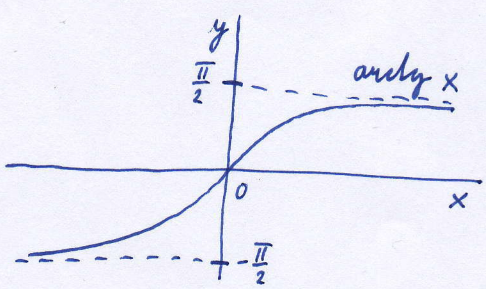
$$= \lim_{x \to \pm\infty} \frac{-11x}{x} = \underline{-11} \qquad \boxed{a_2 : y = 2x - 11}$$

7) $f(x) = \operatorname{arctg} x + x$ $\boxed{D(f) = \mathbb{R}}$ $\rightarrow$ nemá žádné svislé asymptoty

$$k = \lim_{x \to \pm\infty} \frac{\operatorname{arctg} x + x}{x} = \lim_{x \to \pm\infty} \frac{\operatorname{arctg} x}{x} + \lim_{x \to \pm\infty} \frac{x}{x} = \frac{\operatorname{arctg} \pm\infty}{\pm\infty} + 1 = 0 + 1 = \underline{1}$$

$$q_1 = \lim_{x \to \infty} (\operatorname{arctg} x + x - 1 \cdot x) = \lim_{x \to \infty} \operatorname{arctg} x = \underline{\frac{\pi}{2}}$$

$$q_2 = \lim_{x \to -\infty} (\operatorname{arctg} x + x - 1 \cdot x) = \lim_{x \to -\infty} \operatorname{arctg} x = \underline{-\frac{\pi}{2}}$$



$$\boxed{a_1 : y = x + \frac{\pi}{2}}$$
$$\boxed{a_2 : y = x - \frac{\pi}{2}}$$