DÚ 3 – ŘEŠENÍ

1) $y = 2e^x \cdot \sqrt{x^3 - 2x} + \arcsin(2x)$

$$y' = \left(\underbrace{2e^x}_{u} \cdot \underbrace{\sqrt{x^3-2x}}_{v}\right)' + \left(\arcsin(2x)\right)' \quad \text{derivace součtu}$$

$$y' = (2e^x)' \cdot \sqrt{x^3-2x} + 2e^x \cdot \left(\sqrt{x^3-2x}\right)' + \frac{1}{\sqrt{1-(2x)^2}} \cdot (2x)'$$

$$y' = 2e^x \cdot \sqrt{x^3-2x} + 2e^x \cdot \frac{1}{2\sqrt{x^3-2x}} \cdot (3x^2-2) + \frac{1}{\sqrt{1-4x^2}} \cdot 2$$

2) $y = \sqrt{\sin(x-5)} - \operatorname{tg} 7^x + \sin \frac{3\pi}{2}$

$$y' = \left(\sqrt{\sin(x-5)}\right)' - \left(\operatorname{tg} 7^x\right)' + \left(\sin \frac{3\pi}{2}\right)'$$

$$y' = \frac{1}{2\sqrt{\sin(x-5)}} \cdot \left(\sin(x-5)\right)' - \frac{1}{\cos^2 7^x} \cdot (7^x)' + 0$$

$$y' = \frac{1}{2\sqrt{\sin(x-5)}} \cdot \cos(x-5) \cdot 1 - \frac{1}{\cos^2 7^x} \cdot 7^x \cdot \ln 7$$

3) $y = \dfrac{(2x+3)^2}{\sin x}$

$$y' = \frac{\left[(2x+3)^2\right]' \cdot \sin x - (2x+3)^2 \cdot (\sin x)'}{(\sin x)^2}$$

$$y' = \frac{2(2x+3)^1 \cdot (2x+3)' \cdot \sin x - (2x+3)^2 \cdot \cos x}{\sin^2 x}$$

$$y' = \frac{2(2x+3) \cdot 2 \cdot \sin x - (2x+3)^2 \cdot \cos x}{\sin^2 x}$$

4) $y=(2x\cdot \operatorname{tg} x)^3+\sqrt{2\ln x}$

$$y'=3(2x\cdot \operatorname{tg} x)^2\cdot(2x\cdot \operatorname{tg} x)'+\frac{1}{2\sqrt{2\ln x}}\cdot(2\ln x)'$$

$$y'=3(2x\cdot \operatorname{tg} x)^2\cdot\left(2\cdot \operatorname{tg} x+2x\cdot\frac{1}{\cos^2 x}\right)+\frac{1}{2\sqrt{2\ln x}}\cdot 2\cdot\frac{1}{x}$$

5) $y=\sqrt{\log_3 x+\sin^3 x}$

$$y'=\frac{1}{2\sqrt{\log_3 x+\sin^3 x}}\cdot(\log_3 x+\sin^3 x)'$$

$$y'=\frac{1}{2\sqrt{\log_3 x+\sin^3 x}}\cdot\left(\frac{1}{x\cdot\ln 3}+3\sin^2 x\cdot\cos x\right)$$

6) $y=\arccos(3x^2-x)\cdot 5x^3$

$$y'=-\frac{1}{\sqrt{1-(3x^2-x)^2}}\cdot(3\cdot 2x-1)\cdot 5x^3+\arccos(3x^2-x)\cdot 5\cdot 3x^2$$

$$y'=-\frac{1}{\sqrt{1-(3x^2-x)^2}}\cdot(6x-1)\cdot 5x^3+\arccos(3x^2-x)\cdot 15x^2$$

7) $y=e^5\cdot \operatorname{arcctg}(7x)-\operatorname{arctg} e^2$

$$y'=(e^5)'\cdot \operatorname{arcctg}(7x)+e^5\cdot(\operatorname{arcctg}(7x))'-(\operatorname{arctg} e^2)'$$

$$y'=0\cdot \operatorname{arcctg}(7x)+e^5\cdot\left(-\frac{1}{1+(7x)^2}\right)\cdot(7x)'-0$$

$$y'=e^5\cdot\frac{-1}{1+49x^2}\cdot 7$$