DÚ 7 - ŘEŠENÍ

PŘÍMÁ INTEGRACE, PER PARTES

1) $\int (3\sqrt{x} - \frac{5}{x} + 7^x)\,dx = \int 3\sqrt{x}\,dx - \int \frac{5}{x}\,dx + \int 7^x\,dx =$

$= 3\int x^{\frac{1}{2}}\,dx - 5\int \frac{1}{x}\,dx + \int 7^x\,dx = 3 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 5 \cdot \ln|x| + \frac{7^x}{\ln 7} = 3 \cdot \frac{2}{3} x^{\frac{3}{2}} - 5\ln|x| + \frac{7^x}{\ln 7} =$

$= 2\sqrt{x^3} - 5\ln|x| + \frac{7^x}{\ln 7} + C$

2) $\int (\frac{-2}{3x^3} + 6\cos x - 3)\,dx = \int \frac{-2}{3x^3}\,dx + \int 6\cos x\,dx - \int 3\,dx =$

$= -\frac{2}{3}\int \frac{1}{x^3}\,dx + 6\int \cos x\,dx - 3\int dx = -\frac{2}{3}\int x^{-3}\,dx + 6 \cdot \sin x - 3x =$

$= -\frac{2}{3} \cdot \frac{x^{-2}}{-2} + 6\sin x - 3x = \frac{x^{-2}}{3} + 6\sin x - 3x + C$

3) $\int (\frac{\pi}{\sqrt{x}} - 6x - \frac{1}{2})\,dx = \int \frac{\pi}{x^{\frac{1}{2}}}\,dx - \int 6x\,dx - \int \frac{1}{2}\,dx = \pi \int x^{-\frac{1}{2}}\,dx - 6\int x\,dx - \frac{1}{2}\int dx =$

$= \pi \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6\frac{x^2}{2} - \frac{1}{2}x = 2\pi\sqrt{x} - 3x^2 - \frac{x}{2} + C$

4) $\int \cos x\,(1 + \frac{1}{\cos^3 x})\,dx = \int \cos x \cdot 1\,dx + \int \cos x \cdot \frac{1}{\cos^3 x}\,dx =$

$= \int \cos x\,dx + \int \frac{1}{\cos^2 x}\,dx = \sin x + \operatorname{tg} x + C$

5) $\int 3x \cdot \sqrt[3]{x}\,dx = 3\int x^1 \cdot x^{\frac{1}{3}}\,dx = 3\int x^{1+\frac{1}{3}}\,dx = 3\int x^{\frac{4}{3}}\,dx = 3 \cdot \frac{x^{\frac{7}{3}}}{\frac{7}{3}} = 3 \cdot \frac{3}{7} x^{\frac{7}{3}} = \frac{9}{7} x^{\frac{7}{3}} + C$

6) $\int \frac{9}{1+x^2}\,dx = 9\int \frac{1}{1+x^2}\,dx = 9\operatorname{arctg} x + C$

7) $\int -3x \cdot \cos x\,dx = \left| \begin{matrix} u' = \cos x & v = -3x \\ u = \sin x & v' = -3 \end{matrix} \right| = \sin x \cdot (-3x) - \int \sin x \cdot (-3)\,dx =$

$= -3x \cdot \sin x + 3\int \sin x = -3x \cdot \sin x - 3\cos x + C$

8) $\underline{\int(\ln x\cdot x^2+2)\,dx}=\int \ln x\cdot x^2\,dx+2\int dx=\begin{vmatrix} u'=x^2 & v=\ln x \\ u=\frac{x^3}{3} & v=\frac{1}{x}\end{vmatrix}+2\int dx=$

$=\frac{x^3}{3}\cdot\ln x-\int\frac{x^3}{3}\cdot\frac{1}{x}\,dx+2x=\frac{x^3}{3}\ln x-\frac{1}{3}\int x^2\,dx+2x=$

$=\frac{x^3}{3}\ln x-\frac{1}{3}\cdot\frac{x^3}{3}+2x=\underline{\underline{\frac{x^3}{3}\ln x-\frac{x^3}{9}+2x+C}}$

9) $\underline{\int(x^2+3x)e^x}=\underbrace{\int x^2e^x\,dx}_{I_1}+\underbrace{\int 3x\cdot e^x\,dx}_{I_2}=$

$I_1=\int x^2e^x\,dx=\begin{vmatrix} u'=e^x & v=x^2 \\ u=e^x & v'=2x\end{vmatrix}=e^x\cdot x^2-\int e^x\cdot 2x\,dx=e^x\cdot x^2-\begin{vmatrix} u'=e^x & v=2x \\ u=e^x & v'=2\end{vmatrix}=$

$=e^x\cdot x^2-\left(e^x\cdot 2x-\int e^x\cdot 2\,dx\right)=e^x\cdot x^2-e^x\cdot 2x+2\int e^x\,dx=\underline{\underline{e^x\cdot x^2-e^x\cdot 2x+2e^x+C}}$

$I_2=\begin{vmatrix} u'=e^x & v=3x \\ u=e^x & v'=3\end{vmatrix}=e^x\cdot 3x-\int e^x\cdot 3\,dx=\underline{\underline{e^x\cdot 3x-3e^x+C}}$

$I_1+I_2=e^x\cdot x^2-e^x\cdot 2x+2e^x+e^x\cdot 3x-3e^x+C=\underline{\underline{e^x\cdot x^2+e^x\cdot x-e^x+C}}$

10) $\underline{\int\left(\sin x+\frac{1}{x^3}\right)\cdot x\,dx}=\underbrace{\int \sin x\cdot x\,dx}_{I_1}+\underbrace{\int\frac{1}{x^3}\cdot x\,dx}_{I_2}=$

$I_1=\begin{vmatrix} u'=\sin x & v=x \\ u=-\cos x & v'=1\end{vmatrix}=-\cos x\cdot x-\int-\cos x\cdot 1\,dx=-\cos x\cdot x+\int\cos x\,dx=$

$=\underline{\underline{-\cos x\cdot x+\sin x+C}}$

$I_2=\int x^{-3}\cdot x^1\,dx=\int x^{-2}\,dx=\frac{x^{-1}}{-1}=-x^{-1}=\underline{\underline{-\frac{1}{x}+C}}$

$I_1+I_2=\underline{\underline{-\cos x\cdot x+\sin x-\frac{1}{x}+C}}$