ŘEŠENÍ DÚ 6

1) a) $f(x) = \frac{1}{12}x^4 - \frac{7}{6}x^3 + 5x^2 + 12x - 17$

$\boxed{D(f) = \mathbb{R}}$

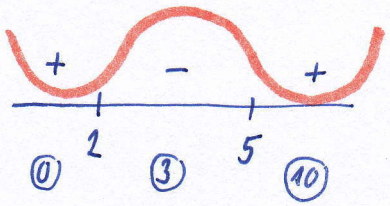
$f'(x) = \frac{1}{12}\cdot 4x^3 - \frac{7}{6}\cdot 3x^2 + 5\cdot 2x + 12 = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 10x + 12$

$f''(x) = \frac{1}{3}\cdot 3x^2 - \frac{7}{2}\cdot 2x + 10 = x^2 - 7x + 10$

$x^2 - 7x + 10 = 0$

$D = 49 - 40 = 9$

$x_{1,2} = \frac{7 \pm 3}{2} \begin{cases} 5 \\ 2 \end{cases}$



KONVEXNÍ: $(-\infty; 2\rangle$ a $\langle 5; \infty)$

KONKÁVNÍ: $\langle 2; 5\rangle$

b) $f(x) = \operatorname{arccotg}(3x+2)$

$f'(x) = -\frac{1}{1+(3x+2)^2}\cdot 3 = -\frac{3}{1+(3x+2)^2}$

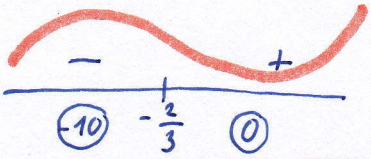
$f''(x) = -\frac{0 - 3\cdot 2(3x+2)^1\cdot 3}{[1+(3x+2)^2]^2} = \frac{18(3x+2)}{\underbrace{[1+(3x+2)^2]^2}_{\oplus}}$

$3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

$\boxed{D(f) = \mathbb{R}}$



KONVEXNÍ: $\langle -\frac{2}{3}; \infty)$

KONKÁVNÍ: $(-\infty; -\frac{2}{3}\rangle$

c) $f(x) = (2x+16)e^x$

$\boxed{D(f) = \mathbb{R}}$

$f'(x) = 2e^x + (2x+16)e^x$

$f''(x) = 2e^x + 2e^x + (2x+16)e^x = e^x(2+2+2x+16) = \underbrace{e^x}_{\oplus}(2x+20)$

nebo:

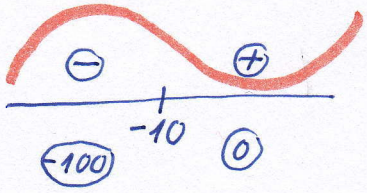
$f'(x) = 2e^x + (2x+16)e^x = e^x(2+2x+16) = e^x(2x+18)$

$f(x) = e^x\cdot(2x+18) + e^x\cdot 2 = e^x(2x+18+2) = \underbrace{e^x}_{\oplus}(2x+20)$

$2x + 20 = 0$

$2x = -20$

$x = -10$



KONVEXNÍ: $\langle -10; \infty)$

KONKÁVNÍ: $(-\infty; -10\rangle$

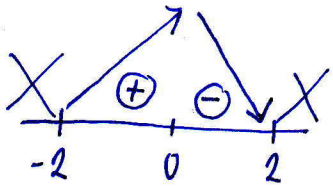
2) a) $f(x) = -\frac{2}{3}x^3 - 3x^2 + 3 \qquad x \in \langle -2; 2 \rangle$

$$f'(x) = -\frac{2}{3}\cdot 3x^2 - 3\cdot 2x = -2x^2 - 6x$$

$$-2x^2 - 6x = 0$$

$$-2x(x+3) = 0$$

N.B.: $0 \qquad -3$



OSTRÉ GLOBÁLNÍ A LOKÁLNÍ MAX: $x = 0$

OSTRÉ GLOBÁLNÍ MIN: $x = 2$

Porovnání:

$$f(-2) = -\frac{2}{3}(-2)^3 - 3\cdot(-2)^2 + 3 = -\frac{2}{3}\cdot(-8) - 3\cdot 4 + 3 = \frac{16}{3} - 12 + 3 = \frac{16}{3} - 9 = \frac{16-27}{3} = -\frac{11}{3} \uparrow$$

$$f(2) = -\frac{2}{3}\cdot 2^3 - 3\cdot 2^2 + 3 = -\frac{2}{3}\cdot 8 - 3\cdot 4 + 3 = -\frac{16}{3} - 12 + 3 = -\frac{16}{3} - 9 = \frac{-16-27}{3} = -\frac{43}{3} \downarrow$$

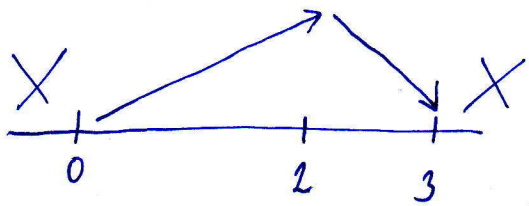
b) $f(x) = \operatorname{arctg}(-3x^2 + 12x - 13) + \operatorname{arctg} 4 \qquad x \in \langle 0; 3 \rangle$

$$f'(x) = \frac{1}{1 + (-3x^2+12x-13)^2}\cdot(-3\cdot 2x + 12) + 0 = \frac{-6x+12}{\underbrace{1 + (-3x^2+12x-13)^2}_{+}}$$

$$-6x + 12 = 0$$

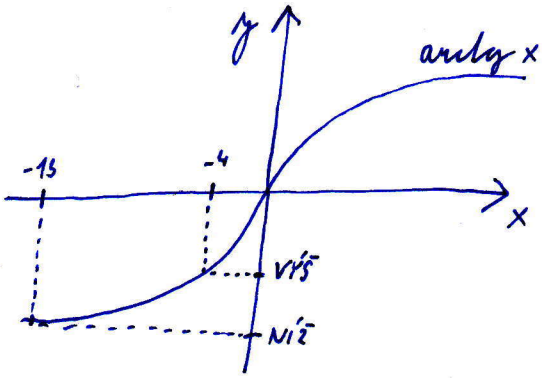
$$6x = 12$$

$$x = 2$$



$f(0) = \operatorname{arctg}(-13) + \operatorname{arctg} 4 \downarrow$

$f(3) = \operatorname{arctg}(-3\cdot 3^2 + 12\cdot 3 - 13) + \operatorname{arctg} 4 = \operatorname{arctg}(-4) + \operatorname{arctg} 4 \uparrow$



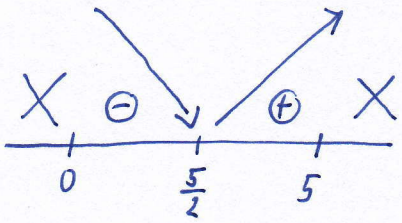
OSTRÉ GLOBÁLNÍ A LOKÁLNÍ MAX: $x = 2$

OSTRÉ GLOBÁLNÍ MIN: $x = 0$

c) $f(x) = 3^{2x^2-10x-3} + 3^5 \qquad x \in \langle 0; 5 \rangle$

$$f'(x) = \underbrace{3^{2x^2-10x-3}}_{\oplus} \cdot \underbrace{\ln 3}_{\oplus} \cdot (4x-10)$$

$$4x - 10 = 0$$
$$4x = 10$$
$$x = \frac{5}{2}$$



$$f(0) = 3^{-3} + 3^5 =$$

$$f(5) = 3^{2\cdot 5^2 - 10\cdot 5 - 3} + 3^5 = 3^{50-50-3} + 3^5 = 3^{-3} + 3^5 =$$

OSTRÉ GLOBÁLNÍ A LOKÁLNÍ MIN: $x = \frac{5}{2}$
NEOSTRÁ GLOBÁLNÍ MAX: $x = 0$, $x = 5$

d) $f(x) = \ln(x^2 + x + 6) + \ln 7 \qquad x \in \langle -2; 2 \rangle$

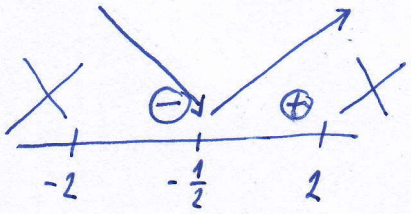
$$f'(x) = \frac{1}{x^2+x+6} \cdot (2x+1)$$

$D = 1 - 24 = -23$

=> Nemá nulový bod

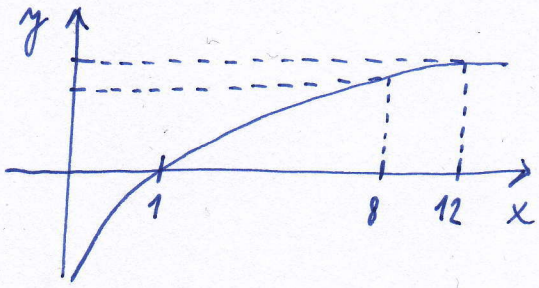
-> ale dosazovat tam pak musím

$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$



$$f(-2) = \ln[(-2)^2 - 2 + 6] + \ln 7 = \ln(4-2+6) + \ln 7 = \ln 8 + \ln 7 \quad \downarrow$$

$$f(2) = \ln(2^2 + 2 + 6) + \ln 7 = \ln(4+2+6) + \ln 7 = \ln 12 + \ln 7 \quad \uparrow$$



OSTRÉ GLOBÁLNÍ A LOKÁLNÍ MIN: $x = -\frac{1}{2}$
OSTRÉ GLOBÁLNÍ MAX: $x = 2$