DÚ 8 - ŘEŠENÍ

SUBSTITUCE

1) $$\int \frac{2x^3}{5x^4+3}\,dx = \left| \begin{array}{l} t=5x^4+3 \\ dt=20x^3\,dx \\ dx=\frac{dt}{20x^3} \end{array} \right| = \int \frac{\cancel{2x^3}}{t}\cdot\frac{dt}{\underset{10}{\cancel{20x^3}}} = \frac{1}{10}\int\frac{dt}{t} = \frac{1}{10}\ln|t| = \frac{1}{10}\ln|5x^4+3|+C$$

2) $$\int \mathrm{cotg}\,x\,dx = \int\frac{\cos x}{\sin x}\,dx = \left| \begin{array}{l} t=\sin x \\ dt=\cos x\,dx \\ dx=\frac{dt}{\cos x} \end{array} \right| = \int\frac{\cancel{\cos x}}{t}\cdot\frac{dt}{\cancel{\cos x}} = \int\frac{dt}{t} = \ln|t| = \ln|\sin x|+C$$

3) $$\int\left[\sin 7x+(5x+4)^9\right]dx = \int\sin 7x\,dx + \int(5x+4)^9\,dx =$$

$$= \left| \begin{array}{l} t=7x \\ dt=7\,dx \\ dx=\frac{dt}{7} \end{array} \right| + \left| \begin{array}{l} s=5x+4 \\ ds=5\,dx \\ dx=\frac{ds}{5} \end{array} \right| = \int\sin t\,\frac{dt}{7} + \int s^9\,\frac{ds}{5} = \frac{1}{7}\int\sin t\,dt + \frac{1}{5}\int s^9\,ds =$$

$$= \frac{1}{7}(-\cos t) + \frac{1}{5}\,\frac{s^{10}}{10} = -\frac{1}{7}\cos t + \frac{s^{10}}{50} = -\frac{1}{7}\cos 7x + \frac{(5x+4)^{10}}{50} + C$$

Pokud si tenhle příklad rozdělíte na dva integrály, můžete v obou substituovat t. Ale jesti ho chcete počítat dohromady, tak se druhý parametr musí jmenovat jinak.

4) $$\int\sqrt{4x^3-8}\cdot 2x^2\,dx = \left| \begin{array}{l} t=4x^3-8 \\ dt=12x^2\,dx \\ dx=\frac{dt}{12x^2} \end{array} \right| = \int\sqrt{t}\cdot\cancel{2x^2}\cdot\frac{dt}{\underset{6}{\cancel{12x^2}}} = \frac{1}{6}\int\sqrt{t}\,dt = \frac{1}{6}\int t^{\frac{1}{2}}\,dt =$$

$$= \frac{1}{6}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{\underset{3}{\cancel{6}}}\cdot\frac{\overset{1}{\cancel{2}}}{3}\cdot t^{\frac{3}{2}} = \frac{1}{9}t^{\frac{3}{2}}+C = \frac{1}{9}(4x^3-8)^{\frac{3}{2}}+C$$

5) $$\int\cos^3 x\cdot\sin x\,dx = \left| \begin{array}{l} t=\cos x \\ dt=-\sin x \\ dx=\frac{-dt}{\sin x} \end{array} \right| = \int t^3\cdot\cancel{\sin x}\cdot\frac{-dt}{\cancel{\sin x}} = -\int t^3\,dt = -\frac{t^4}{4} =$$

$$= -\frac{\cos^4 x}{4}+C$$

6) $\int x \cdot e^{3x^2} dx = \left| \begin{array}{l} t = 3x^2 \\ dt = 6x\,dx \\ dx = \frac{dt}{6x} \end{array} \right| = \int \cancel{x} \cdot e^t \cdot \frac{dt}{6\cancel{x}} = \frac{1}{6} \int e^t dt = \frac{1}{6} e^t = \frac{1}{6} e^{3x^2} + C$

7) $\int e^x \cdot \cos e^x dx = \left| \begin{array}{l} t = e^x \\ dt = e^x dx \\ dx = \frac{dt}{e^x} \end{array} \right| = \int \cancel{e^x} \cdot \cos t \cdot \frac{dt}{\cancel{e^x}} = \int \cos t\, dt = \sin t =$

$= \sin e^x + C$

8) $\int \frac{2}{\sin^2(3x+1)} dx = \left| \begin{array}{l} t = 3x+1 \\ dt = 3\,dx \\ dx = \frac{dt}{3} \end{array} \right| = \int \frac{2}{\sin^2 t} \frac{dt}{3} = \frac{2}{3} \int \frac{dt}{\sin^2 t} = \frac{2}{3} \cdot (-\operatorname{cotg} t) =$

$= -\frac{2}{3} \operatorname{cotg}(3x+1) + C$