ŘEŠENÍ - 9

NEURČITÝ INTEGRÁL

1) $\int x \left(\frac{1}{\sin^2(x^2+9)} + e^x\right) dx =$

$$= \underbrace{\int \frac{x}{\sin^2(x^2+9)}\,dx}_{I_1} + \underbrace{\int x\cdot e^x\,dx}_{I_2} = \boxed{-\frac{1}{2}\operatorname{cotg}(x^2+9) + e^x\cdot x - e^x + C}$$

$$I_1 = \left|\begin{matrix} t = x^2+9 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x}\end{matrix}\right| = \int \frac{\cancel{x}}{\sin^2 t}\cdot\frac{dt}{2\cancel{x}} = \frac{1}{2}\int\frac{dt}{\sin^2 t} = \frac{1}{2}\cdot(-\operatorname{cotg} t) = -\frac{1}{2}\operatorname{cotg}(x^2+9)+C$$

$$I_2 = \int x\cdot e^x\,dx = \left|\begin{matrix} u' = e^x & v = x \\ u = e^x & v' = 1\end{matrix}\right| = e^x\cdot x - \int e^x\cdot 1\,dx = e^x\cdot x - e^x + C$$

2) $\int x(\cos x + \sqrt{x})\,dx =$

$$= \underbrace{\int x\cdot\cos x\,dx}_{I_1} + \underbrace{\int x\cdot\sqrt{x}\,dx}_{I_2} = \boxed{\sin x\cdot x + \cos x + \frac{2}{5}x^{\frac{5}{2}} + C}$$

$$I_1 = \left|\begin{matrix} u' = \cos x & v = x \\ u = \sin x & v' = 1\end{matrix}\right| = \sin x\cdot x - \int \sin x\cdot 1\,dx = \sin x\cdot x + \cos x + C$$

$$I_2 = \int x\cdot x^{\frac{1}{2}}\,dx = \int x^{1+\frac{1}{2}}\,dx = \int x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2}{5}x^{\frac{5}{2}} + C$$

3) $\int \cos x\left(x + (\sin x + 2)^7\right)dx =$

$$= \underbrace{\int \cos x\cdot x\,dx}_{I_1} + \underbrace{\int \cos x(\sin x+2)^7\,dx}_{I_2} = \boxed{\sin x\cdot x + \cos x + \frac{(\sin x+2)^8}{8} + C}$$

$$I_1 = \left|\begin{matrix} u' = \cos x & v = x \\ u = \sin x & v' = 1\end{matrix}\right| = \sin x\cdot x - \int \sin x\,dx = \sin x\cdot x + \cos x + C$$

$$I_2 = \left|\begin{matrix} t = \sin x + 2 \\ dt = \cos x\,dx \\ dx = \frac{dt}{\cos x}\end{matrix}\right| = \int \cancel{\cos x}\cdot t^7\frac{dt}{\cancel{\cos x}} = \frac{t^8}{8} = \frac{(\sin x+2)^8}{8} + C$$

4) $\int \operatorname{arccotg}(3x)\,dx =$

$$= \int 1\cdot \operatorname{arccotg}(3x)\,dx = \left| \begin{array}{ll} u' = 1 & v = \operatorname{arccotg}(3x) \\ u = x & v' = \dfrac{-1}{1+(3x)^2}\cdot 3 \end{array} \right| =$$

$$= x\cdot \operatorname{arccotg}(3x) - \int x\cdot \frac{-1}{1+(3x)^2}\cdot 3\,dx = x\cdot \operatorname{arccotg}(3x) + \int \frac{3x}{1+9x^2} =$$

$$= x\cdot \operatorname{arccotg}(3x) + \left| \begin{array}{l} t = 1+9x^2 \\ dt = 18x\,dx \\ dx = \dfrac{dt}{18x} \end{array} \right| = x\cdot \operatorname{arccotg}(3x) + \int \frac{\cancel{3x}}{t}\cdot \frac{dt}{\underset{6}{\cancel{18x}}} =$$

$$= x\cdot \operatorname{arccotg}(3x) + \frac{1}{6}\int \frac{dt}{t} = x\cdot \operatorname{arccotg}(3x) + \frac{1}{6}\ln|t| =$$

$$= \boxed{x\cdot \operatorname{arccotg}(3x) + \frac{1}{6}\ln|1+9x^2| + C}$$

5) $\int \cos x\left(x + \frac{1}{\sin x + 2}\right)dx =$

$$= \underbrace{\int \cos x\cdot x\,dx}_{I_1} + \underbrace{\int \cos x\cdot \frac{1}{\sin x+2}\,dx}_{I_2} = \boxed{\sin x\cdot x + \cos x + \ln|\sin x + 2| + C}$$

$$I_1 = \sin x\cdot x + \cos x + C$$

$$I_2 = \left| \begin{array}{l} t = \sin x + 2 \\ dt = \cos x\,dx \\ dx = \dfrac{dt}{\cos x} \end{array} \right| = \int \cancel{\cos x}\cdot \frac{1}{t}\cdot \frac{dt}{\cancel{\cos x}} = \int \frac{1}{t}\,dt = \ln|t| = \ln|\sin x + 2| + C$$

6) $\int x \cdot (\ln x + (x^2+1)^7)\,dx =$

$$= \underbrace{\int x \cdot \ln x \, dx}_{I_1} + \underbrace{\int x \cdot (x^2+1)^7 dx}_{I_2} = \boxed{\frac{x^2}{2}\ln x - \frac{x^2}{4} + \frac{(x^2+1)^8}{16} + C}$$

$$I_1 = \begin{vmatrix} u' = x & v = \ln x \\ u = \frac{x^2}{2} & v' = \frac{1}{x} \end{vmatrix} = \frac{x^2}{2}\cdot \ln x - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2} =$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$I_2 = \begin{vmatrix} t = x^2+1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{vmatrix} = \int \cancel{x}\cdot t^7 \frac{dt}{2\cancel{x}} = \frac{1}{2}\int t^7 dt = \frac{1}{2}\cdot\frac{t^8}{8} = \frac{t^8}{16} = \frac{(x^2+1)^8}{16} + C$$

7) $\int x\left(\sqrt{x^2+4} + \cos x\right)dx =$

$$= \underbrace{\int x \cdot \sqrt{x^2+4}\,dx}_{I_1} + \underbrace{\int x \cdot \cos x\,dx}_{I_2} = \boxed{\frac{1}{3}(x^2+4)^{\frac{3}{2}} + \sin x \cdot x + \cos x + C}$$

$$I_1 = \begin{vmatrix} t = x^2+4 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{vmatrix} = \int \cancel{x}\cdot\sqrt{t}\frac{dt}{2\cancel{x}} = \frac{1}{2}\int t^{\frac{1}{2}}dt = \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\,t^{\frac{3}{2}} = \frac{1}{3}(x^2+4)^{\frac{3}{2}} + C$$

$$I_2 = \sin x \cdot x + \cos x + C$$

8) $\int e^{\sqrt{3+x}} =$

$$= \begin{vmatrix} t = \sqrt{3+x} \;/^2 \\ t^2 = 3+x \\ 2t\,dt = dx \end{vmatrix} = \int e^t 2t\,dt = 2\int e^t \cdot t\,dt = 2\begin{vmatrix} u' = e^t & v = t \\ u = e^t & v' = 1 \end{vmatrix} =$$

$$= 2\left(e^t\cdot t - \int e^t \cdot 1\,dt\right) = 2e^t\cdot t - 2\int e^t dt = 2e^t\cdot t - 2e^t =$$

$$= \boxed{2e^{\sqrt{3+x}}\cdot\sqrt{3+x} - 2e^{\sqrt{3+x}} + C}$$