

$$1) \underline{y'' + 4y = (10x+1)e^{-x}}$$

$$y'' + 4y = 0$$

$$k^2 + 4 = 0$$

$$k^2 = -4$$

$$\underline{k = \pm 2i}$$

$$y = C_1 e^0 \cos(2x) + C_2 e^0 \sin(2x) + v(x)$$

$$\underline{y = C_1 \cos(2x) + C_2 \sin(2x) + v(x)}$$

$$v(x) = ax e^{-x} + b e^{-x}$$

$$\begin{aligned} v'(x) &= a e^{-x} + ax(-e^{-x}) + b(-e^{-x}) = \\ &= a e^{-x} - ax e^{-x} - b e^{-x} \end{aligned}$$

$$\begin{aligned} v''(x) &= a(-e^{-x}) - a e^{-x} - ax(-e^{-x}) - b(-e^{-x}) = \\ &= -a e^{-x} - a e^{-x} + ax e^{-x} + b e^{-x} = \\ &= \underline{-2a e^{-x} + ax e^{-x} + b e^{-x}} \end{aligned}$$

$$\underline{-2a e^{-x} + ax e^{-x} + b e^{-x} + 4ax e^{-x} + 4b e^{-x} = 10x e^{-x} + e^{-x}}$$

$$x e^{-x}: \quad a + 4a = 10$$

$$5a = 10$$

$$\underline{a = 2}$$

$$e^{-x}: \quad -2 \cdot 2 + b + 4b = 1$$

$$-4 + 5b = 1$$

$$5b = 5$$

$$\underline{b = 1}$$

$$\underline{y = C_1 \cos(2x) + C_2 \sin(2x) + 2x e^{-x} + e^{-x}}$$

$$2) \underline{-y'' - 7y' - 6y = -21 \sin(3x) - 3 \cos(3x)}$$

$$-y'' - 7y' - 6y = 0$$

$$-k^2 - 7k - 6 = 0$$

$$k^2 + 7k + 6 = 0$$

$$(k+1)(k+6) = 0$$

$$\underline{k_1 = -1}$$

$$\underline{k_2 = -6}$$

$$\underline{y = C_1 \cdot e^{-x} + C_2 \cdot e^{-6x} + v(x)}$$

$$v(x) = a \sin(3x) + b \cos(3x)$$

$$v'(x) = a \cdot \cos(3x) \cdot 3 + b(-\sin(3x)) \cdot 3 =$$

$$= 3a \cos(3x) - 3b \sin(3x)$$

$$v''(x) = 3a(-\sin(3x)) \cdot 3 - 3b \cdot \cos(3x) \cdot 3 =$$

$$\underline{= -9a \sin(3x) - 9b \cos(3x)}$$

$$-[-9a \sin(3x) - 9b \cos(3x)] - 7[3a \cos(3x) - 3b \sin(3x)] - 6[a \sin(3x) + b \cos(3x)] = -21 \sin(3x) - 3 \cos(3x)$$

$$\underline{9a \sin(3x) + 9b \cos(3x) - 21a \cos(3x) + 21b \sin(3x) - 6a \sin(3x) - 6b \cos(3x) = -21 \sin(3x) - 3 \cos(3x)}$$

$$\sin(3x): 9a + 21b - 6a = -21$$

$$a + 7 \cdot (-1) = -7$$

$$\cos(3x): 9b - 21a - 6b = -3$$

$$a - 7 = -7$$

$$3a + 21b = -21 \quad | :3$$

$$\underline{a = 0}$$

$$\underline{-21a + 3b = -3 \quad | :3}$$

$$a + 7b = -7 \quad | \cdot 7$$

$$\underline{-7a + b = -1}$$

$$7a + 49b = -49$$

$$\underline{-7a + b = -1}$$

$$50b = -50$$

$$\underline{b = -1}$$

$$\underline{y = C_1 \cdot e^{-x} + C_2 \cdot e^{-6x} - \cos(3x)}$$

$$3) \underline{y'' - 3y' - 4y = 10e^{-x}}$$

$$y'' - 3y' - 4y = 0$$

$$k^2 - 3k - 4 = 0$$

$$(k-4)(k+1) = 0$$

$$\underline{k_1 = 4}$$

$$\underline{k_2 = -1}$$

$$\underline{y = C_1 e^{4x} + C_2 e^{-x} + v(x)}$$

$$v(x) = axe^{-x} + be^{-x}$$

$$v'(x) = ae^{-x} - axe^{-x} - be^{-x}$$

$$v''(x) = -ae^{-x} - ae^{-x} + axe^{-x} + be^{-x} =$$

$$\underline{= -2ae^{-x} + axe^{-x} + be^{-x}}$$

$$\underline{-2ae^{-x} + axe^{-x} + be^{-x} - 3ae^{-x} + 3axe^{-x} + 3be^{-x} - 4axe^{-x} - 4be^{-x} = 10e^{-x}}$$

$$xe^{-x}: a + 3a - 4a = 0$$

$$0a = 0$$

$$e^{-x}: -2a + b - 3a + 3b - 4b = 10$$

$$-5a + 0b = 10$$

$$-5a = 10$$

$$\underline{a = -2}$$

→ 0 b mi nevím a proto ho

do řešení nedávám.

$$\underline{y = C_1 e^{4x} + C_2 e^{-x} - 2xe^{-x}}$$

$$4) \underline{y'' - 4y' + 4y = 4x^2 + 2x + 2}$$

$$y'' - 4y' + 4y = 0$$

$$k^2 - 4k + 4 = 0$$

$$(k - 2)^2 = 0$$

$$\underline{k = 2}$$

$$v_{(x)} = ax^2 + bx + c$$

$$v'_{(x)} = 2ax + b$$

$$\underline{v''_{(x)} = 2a}$$

$$\underline{y = C_1 \cdot e^{2x} + C_2 \cdot e^{2x} \cdot x + v_{(x)}}$$

$$2a - 4(2ax + b) + 4(ax^2 + bx + c) = 4x^2 + 2x + 2$$

$$\underline{2a - 8ax - 4b + 4ax^2 + 4bx + 4c = 4x^2 + 2x + 2}$$

$$x^2: 4a = 4$$

$$\underline{a = 1}$$

$$x: -8 \cdot 1 + 4b = 2$$

$$4b = 10$$

$$\underline{b = \frac{5}{2}}$$

$$x^0: 2 \cdot 1 - 4 \cdot \frac{5}{2} + 4c = 2$$

$$2 - 10 + 4c = 2$$

$$4c = 10$$

$$\underline{c = \frac{5}{2}}$$

$$\underline{y = C_1 e^{2x} + C_2 e^{2x} \cdot x + x^2 + \frac{5}{2}x + \frac{5}{2}}$$

$$5) \underline{-y'' - 3y' + 4y = 18 \cos(3x) - 26 \sin(3x)}$$

$$-y'' - 3y' + 4y = 0$$

$$-k^2 - 3k + 4 = 0$$

$$k^2 + 3k - 4 = 0$$

$$(k+4)(k-1) = 0$$

$$\underline{k_1 = -4}$$

$$\underline{k_2 = 1}$$

$$\underline{y = C_1 e^{-4x} + C_2 e^x + v(x)}$$

$$v(x) = a \sin(3x) + b \cos(3x)$$

$$v'(x) = 3a \cos(3x) - 3b \sin(3x)$$

$$\underline{v''(x) = -9a \sin(3x) - 9b \cos(3x)}$$

$$\underline{9a \sin(3x) + 9b \cos(3x) - 9a \cos(3x) + 9b \sin(3x) + 4a \sin(3x) + 4b \cos(3x) = 18 \cos(3x) - 26 \sin(3x)}$$

$$\sin(3x): 9a + 9b + 4a = -26$$

$$\cos(3x): 9b - 9a + 4b = 18$$

$$13a + 9b = -26 \quad | \cdot 9$$

$$\underline{-9a + 13b = 18 \quad | \cdot 13}$$

$$117a + 81b = -234$$

$$-117a + 169b = 234$$

$$250b = 0$$

$$\underline{b = 0}$$

26	13	18
<u>· 9</u>	<u>· 13</u>	<u>· 13</u>
54	39	54
<u>18</u>	<u>13</u>	<u>18</u>
234	169	234

$$-9a + 13 \cdot 0 = 18$$

$$-9a = 18$$

$$\underline{a = -2}$$

$$\underline{y = C_1 e^{-4x} + C_2 e^x - 2 \sin(3x)}$$

$$6) \underline{y'' - 4y' + 5y = 3x e^{2x}}$$

$$y'' - 4y' + 5y = 0$$

$$k^2 - 4k + 5 = 0$$

$$D = 16 - 4 \cdot 5 = -4$$

$$\sqrt{D} = \sqrt{-4} = 2i$$

$$x_{1,2} = \frac{4 \pm 2i}{2} \begin{cases} \underline{\underline{2+i}} \\ \underline{\underline{2-i}} \end{cases}$$

$$\underline{y = C_1 e^{2x} \sin x + C_2 e^{2x} \cos x + v(x)}$$

$$v(x) = ax e^{2x} + b e^{2x}$$

$$v'(x) = a e^{2x} + ax e^{2x} \cdot 2 + b e^{2x} \cdot 2 =$$

$$= a e^{2x} + 2ax e^{2x} + 2b e^{2x}$$

$$v''(x) = a e^{2x} \cdot 2 + 2a e^{2x} + 2ax e^{2x} \cdot 2 + 2b e^{2x} \cdot 2 =$$

$$= 2a e^{2x} + 2a e^{2x} + 4ax e^{2x} + 4b e^{2x} =$$

$$= 4a e^{2x} + 4ax e^{2x} + 4b e^{2x}$$

$$\underline{4a e^{2x} + 4ax e^{2x} + 4b e^{2x} - 4a e^{2x} - 8ax e^{2x} - 8b e^{2x} + 5ax e^{2x} + 5b e^{2x} = 3x e^{2x}}$$

$$x e^{2x}: 4a - 8a + 5a = 3$$

$$\underline{a = 3}$$

$$e^{2x}: 4a + 4b - 4a - 8b + 5b = 0$$

$$\underline{b = 0}$$

$$\underline{y = C_1 e^{2x} \sin x + C_2 e^{2x} \cos x + 3x e^{2x}}$$